

Quark–Quark Forces in Quantum Chromodynamics*

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Abstract

By single-time reduction technique of Bethe-Salpeter formalism for two-fermion systems analytical expressions for the quasipotential of quark-quark interactions in QCD have been obtained in one-gluon exchange approximation. The influence of infrared singularities of gluon Green's functions on the character of quark-quark forces in QCD has been investigated. The way the asymptotic freedom manifests itself in terms of two-quark interaction quasipotential in quantum chromodynamics is shown. Consistent relativistic consideration of quark interaction problem by single-time reduction technique in QFT allows one to establish a nontrivial energy dependence of the two-quark interaction quasipotential. As a result of the energy dependence of the interaction quasipotential, the character of the forces changes qualitatively during the transition from the discrete spectrum (the region of the negative values of the binding energy) to the continuous spectrum (that of the positive values of the binding energy): the smooth behaviour of the interaction quasipotential in the discrete spectrum goes into the oscillation in the continuous spectrum. This result gives a visual physical picture where the oscillations may be interpreted as a manifestation of a quasicrystal structure of the vacuum.

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1 Introduction

The investigation of relativistic systems interaction properties is one of the central point in elementary particle physics and the physics of atomic nucleus. Two-fermion systems are the very tool with the help of which we study and check our notions about fundamental forces acting in Nature. The hydrogen atom is the most well-known example of a two-fermion system. Figuratively speaking one may say that the hydrogen atom was the laboratory where has been created, checked and studied one of the wonderful physical theory of the XX century, i.e., quantum mechanics. The first checks of quantum electrodynamics were also carried out on the two-fermion systems such as hydrogen atom, positronium, muonium, etc. [1]. The study of deuteron as a system consisting of two nucleons allowed one to go considerably forth in our understanding of nuclear forces [2]. The problem of nucleon-nucleon interaction is still one of the fundamental problems in nuclear physics.

The observation of the J/Ψ and Υ mesons consisting of $c\bar{c}$ and $b\bar{b}$ quarks stimulated our interest in a theoretical description of the properties of two-fermion systems. At present in the framework of the local quantum field theory we have a fundamental gauge model for strong interactions known as quantum chromodynamics (QCD). The problem of describing the spectroscopy of hadronic states, as bound states of quark and gluon fields, remains to be one of the important but unsolved problems of QCD.

At the same time the spectroscopy of the J/Ψ and Υ particle families finds its excellent description in the framework of the potential models with phenomenological potentials [3,4]. In this, a very interesting fact has been revealed, the spectroscopy of quarkonium systems has many features similar to those of the spectroscopy of hydrogen atom and positronium. This is the reason why the quarkonium system where the interaction between quarks is given by Coulomb-like potential, added with a term linearly growing with distance, is sometimes declared to be a "hydrogen atom" for strong-interacting systems.

The success of the potential models in describing the spectroscopy of quarkonium systems may be thought to be not accidental and in order to understand why it is so, one should, first of all, clarify what is the status of the potential models in the framework of QCD. The solution of this problem would allow one to establish the connection between the fundamental theory and experiment right at the point where at present we have confrontation.

Note that indicated point of the confrontation between theory and experiment is an essentially nonperturbative region.

In a more general case this problem may be formulated as a problem of calculating the interaction potential between quarks proceeding from the first principles. In the case the problem be solved one would manage to predict all the properties of quark systems proceeding from the fundamental QCD Lagrangian. Here great hopes are set on the calculations based on the lattice methods and large efforts are undertaken in this direction [5-7].

In the present report we are going to show that there exist a simpler and more consistent way to solve this problem, which is based on the single-time formalism in quantum field theory (QFT). Here we will present the results of our last works where the problem of calculating the quark-quark forces in QCD has been considered. We have used the single-time formalism in QFT as a tool in the investigation of the problem.

2 Necessary Information on Single-Time Reduction Technique

As usually we shall introduce the Bethe-Salpeter wave function of the composite two-fermion quark-antiquark system with the help of the matrix element

$$\Phi_i^j(x_1 x_2) = \langle 0 | T(\Psi_i(x_1) \bar{\Psi}^j(x_2)) | \Phi \rangle,$$

where $\Psi_i(x_i)$ are Heisenberg operators of the quark fields, $| \Phi \rangle$ is a normalized vector of the bound state of quark-antiquark system. Heisenberg operators have an index, which in the case of QCD is complicated and is a set of three indices: $i = (\alpha_1, f_1, c_1)$, $j = (\alpha_2, f_2, c_2)$, where α is the spinor index, f is the flavour index, and c is the colour index. The single-time wave function of quark-antiquark system is defined with the equation [8]

$$\begin{aligned} \tilde{\Psi}(n\tau | x_1 x_2) &= \\ &= \frac{1}{i^2} \iint_{n\zeta_1 = \tau = n\zeta_2} S_1^{(-)}(x_1 + n\tau - \zeta_1) d\hat{\sigma}_{\zeta_1} \Phi(\zeta_1 \zeta_2) d\hat{\sigma}_{\zeta_2} S_2^{(+)}(\zeta_2 - n\tau - x_2). \end{aligned} \quad (1)$$

In the R.H.S. of Eq. (1) we have not only integration over space-like surface but summation over indices, which we do not write down explicitly. Here

one should bear in mind that

$$[S_1^{(-)}(x)]_i^{i'} = \delta_f^{f'} \delta_c^{c'} [S_1^{(-)}]_\alpha^{\alpha'}, \quad [S_2^{(+)}]_{j'}^j = \delta_f^{f'} \delta_c^{c'} [S_2^{(+)}]_{\beta'}^{\beta},$$

where $[S_1^{(-)}(x)]_\alpha^{\alpha'}$, $[S_2^{(+)}(x)]_{\beta'}^{\beta}$ are frequency parts of the one-particle causality Green's function of the spinor fields and

$$[d\hat{\sigma}_{\zeta_1}]_i^{i'} = \delta_f^{f'} \delta_c^{c'} [\gamma_\mu]_\alpha^{\alpha'} d\sigma_{\zeta_1}^\mu, \quad [d\hat{\sigma}_{\zeta_2}]_{j'}^{j'} = \delta_f^{f'} \delta_c^{c'} [\gamma_\mu]_{\beta'}^{\beta'} d\sigma_{\zeta_2}^\mu,$$

where $d\sigma_\zeta^\mu$ is a differential element of a flat space-like surface at the point ζ . The surface is given by the Eq. $n\zeta = \tau$, where $n^\mu (n^2 = 1)$ is a unit time-like vector of the normal to the given flat hypersurface.

The momentum representation for single-time wave function (1) is introduced with the following integral transformation [8]

$$\begin{aligned} \Psi(n\tau \mid \tilde{p}_1 \sigma_1 \tilde{p}_2 \sigma_2) &= \\ &= \iint \bar{u}(x_1 \mid \vec{p}_1 \sigma_1) d\hat{\sigma}_{x_1} \tilde{\Psi}(n\tau \mid x_1 x_2) d\hat{\sigma}_{x_2} v(x_2 \mid \vec{p}_2 \sigma_2), \end{aligned} \quad (2)$$

where in the R.H.S. of Eq. (2) the integration is carried out over some space-like surfaces and one can easily verify that the result of such integration does not depend on the choice of these surfaces. Summation over the indices, which is not explicitly given here, however implied. The index σ is a compound one and is a set of three indices $\sigma = (\sigma_\alpha, \sigma_f, \sigma_c)$. The functions u and v are one-particle wave functions of quark and antiquark, respectively, which satisfy the normalization conditions

$$\begin{aligned} \int \bar{u}(x \mid \vec{p}\sigma) d\hat{\sigma}_x u(x \mid \vec{k}\sigma') &= \int \bar{v}(x \mid \vec{p}\sigma) d\hat{\sigma}_x v(x \mid \vec{k}\sigma') = \\ &= 2E(\vec{p}) \delta^3(\vec{p} - \vec{k}) \delta_{\sigma\sigma'} \end{aligned}$$

and of completeness

$$\begin{aligned} \sum_\sigma \int d\mu(\vec{p}) u(x \mid \vec{p}\sigma) \bar{u}(y \mid \vec{p}\sigma) &= \frac{1}{i} S^{(-)}(x - y), \\ \sum_\sigma \int d\mu(\vec{p}) v(x \mid \vec{p}\sigma) \bar{v}(y \mid \vec{p}\sigma) &= \frac{1}{i} S^{(+)}(x - y), \end{aligned}$$

where $d\mu(\vec{p})$ is an invariant measure in the momentum space (an element of the one-particle phase volume)

$$d\mu(\vec{p}) = (2\sqrt{m^2 + \vec{p}^2})^{-1} d^3\vec{p}.$$

Everywhere tilde above the momentum implies, that the given momentum lies on the mass shell $\tilde{p}_i^2 = m_i^2$.

Introduce also the Fourier transformation of the single-time wave function over the variable τ

$$\Psi(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) = \int_{-\infty}^{\infty} d\tau \exp(iM\tau) \Psi(n\tau \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2).$$

For the single-time wave function thus defined in ref.[8] we obtained a three-dimensional dynamic equation, which has the form

$$\begin{aligned} \Psi(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2) &= (n\tilde{p}_1 + n\tilde{p}_2 - M)^{-1} \sum_{\lambda_1\lambda_2} \iint d\mu_1(\vec{k}_1) d\mu(\vec{k}_2) \times \\ &\times V(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2; \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) \Psi(nM \mid \tilde{k}_1\lambda_1\tilde{k}_2\lambda_2). \end{aligned} \quad (3)$$

The function V in the R.H.S. of Eq. (3) describes the quark-antiquark interaction and is defined with the equation

$$\begin{aligned} V(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2; \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) &= T(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2; \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) \\ &- \sum_{\sigma'_1\sigma'_2} \iint d\mu(\vec{p}'_1) d\mu(\vec{p}'_2) (n\tilde{p}'_1 + n\tilde{p}'_2 - M)^{-1} \times \\ &\times V(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2; \tilde{p}'_2\sigma'_2\tilde{p}'_1\sigma'_1) T(nM \mid \tilde{p}'_1\sigma'_1\tilde{p}'_2\sigma'_2; \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1). \end{aligned} \quad (4)$$

where the function T is given by the equality

$$\begin{aligned} T(nM \mid \tilde{p}_1\sigma_1\tilde{p}_2\sigma_2; \tilde{k}_2\lambda_2\tilde{k}_1\lambda_1) &= in_0\delta^3(\vec{P}_M - \vec{K}_M) \\ \bar{u}(\vec{p}_1\sigma_1)v(\vec{p}_2\sigma_2)(2\pi)^{-5} \int_{-\infty}^{\infty} d\alpha &\left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\ &\times \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \times \\ &\times \tilde{R}(p - \alpha n; k - \beta n \mid K_M) u(\vec{k}_1\lambda_1) \bar{v}(\vec{k}_2\lambda_2), \end{aligned} \quad (5)$$

where $R(p; k | K)$ is the Fourier-image of the VEV of the forth order radiation operator

$$\begin{aligned}
\tilde{R}(p_1 p_2; k_2 k_1) &= (2\pi)^4 \delta^4(P - K) \tilde{R}(p; k | K), \\
\tilde{R}(p_1 p_2; k_2 k_1) &= \int dx_1 dx_2 dy_1 dy_2 \tilde{R}^{(4)}(x_1 x_2; y_2 y_1) \times \\
&\quad \times \exp(ip_1 x_1 + ip_2 x_2 - ik_1 y_1 - ik_2 y_2), \\
\tilde{R}^{(4)}(x_1 x_2; y_2 y_1) &= R^{(4)}(x_1 y_2; x_2 y_1), \\
R^{(4)}(x_1 x_2; y_2 y_1) &= \frac{1}{i^2} < 0 | \frac{\delta^4 S}{\delta \bar{\psi}(x_1) \delta \bar{\psi}(x_2) \delta \psi(y_2) \delta \psi(y_1)} S^+ | 0 > .
\end{aligned}$$

In another words the function $R^{(4)}$ is the current Green's function because it coincides with the VEV of chronological product of fermion currents up to quasilocal terms (for details refer to [8,9]). The following notations are used in the R.H.S. of Eq.(5)

$$\begin{aligned}
P &= \tilde{p}_1 + \tilde{p}_2, \quad p = \frac{1}{2}(\tilde{p}_1 - \tilde{p}_2), \quad P_M = P - \Delta n, \\
K &= \tilde{k}_1 + \tilde{k}_2, \quad k = \frac{1}{2}(\tilde{k}_1 - \tilde{k}_2), \quad K_M = K - \Delta' n, \\
\Delta &= nP - M, \quad \Delta' = nK - M.
\end{aligned}$$

Besides a convolution over the indices, not written down explicitly, is implied in the R.H.S of Eq. (5).

It is worth stress that the Schrödinger structure of the dynamic equation for single-time wave function arises as a consequence of the causality structure of local quantum field theory. When deriving the three-dimensional dynamic equations we did not bare in mind any concrete model of the QFT, but used its most general properties. Therefore the dynamic equation (3) may serve a reliable foundation for phenomenological investigation of relativistic two-fermion systems. At the same time the single-time reduction technique may serve as effective tool to investigate any particular quantum field theory model. As such a particular model, the gauge model was picked up known as quantum chromodynamics. On the one hand, this choice was partially due to that, quantum chromodynamics claims to describe the hadronic sector of the so-called Standard Model and, on the other hand, includes, as a special case, another gauge model called quantum electrodynamics (QED), the latter describing excellently electromagnetic interactions in particle physics.

The calculation of two-quark interaction quasipotential in QCD was carried out in three stages. The one-gluon approximation has been used in the first stage. Afterwards the influence of infrared singularities of gluon Green's functions on the character of two-quark forces in QCD has been investigated. Finally the way the asymptotic freedom manifests itself in terms of two-quark interaction quasipotential in quantum chromodynamics was shown.

3 One-Gluon Exchange Approximation in QCD

Using the QCD Lagrangian structure, we obtain for the function \tilde{R} in the R.H.S. of Eq. (5) in the second order over the coupling constant the representation of the form (one-gluon exchange approximation)

$$\tilde{R}(p; k|K) = ig^2[t_{(1)}^a][t_{(2)}^b]\gamma_{(1)}^\mu\gamma_{(2)}^\nu D_{\mu\nu}^{(0)ab}(p-k), \quad (6)$$

where t^a are the generators of the gauge transformations, $D_{\mu\nu}^{(0)ab}$ is a propagator of the massless vector gluon, for which we use a standard expression (in covariant gauge)[9]

$$D_{\mu\nu}^{(0)ab}(q) = \delta^{ab} \frac{-1}{q^2 + i\varepsilon} (g_{\mu\nu} + (d^{(0)} - 1) \frac{q_\mu q_\nu}{q^2 + i\varepsilon}). \quad (7)$$

Here $d^{(0)}$ is a parameter, which fixes the gauge of gluon field. Substituting the representation (6) for the function \tilde{R} in R.H.S. of (5) for the quark-antiquark interaction quasipotential we get [10,11]

$$\begin{aligned} V_{q_1\bar{q}_2}(nM|\tilde{p}_1\sigma_{i_1}\tilde{p}_2\sigma_{i_2}; \tilde{k}_2\lambda_{j_2}\tilde{k}_1\lambda_{j_1}) &= n_0\delta^3(\vec{P}_M - \vec{K}_M)\delta_{\sigma_{f_1}}^{\lambda_{f_1}}\delta_{\sigma_{f_2}}^{\lambda_{f_2}} \sum_a [t_{(1)}^a]_{\sigma_{c_1}}^{\lambda_{c_1}} [t_{(2)}^a]_{\lambda_{c_2}}^{\sigma_{c_2}} \times \\ &\times \left[\bar{u}(\vec{p}_1\sigma_1)\gamma^\mu u(\vec{k}_1\lambda_1)\bar{v}(\vec{k}_2\lambda_2)\gamma_\mu v(\vec{p}_2\sigma_2)A^{(0)}(nM|\tilde{p}_1\tilde{p}_2; \tilde{k}_1\tilde{k}_2) + \right. \\ &\left. + (d^{(0)} - 1)\bar{u}(\vec{p}_1\sigma_1)(n\gamma)u(\vec{k}_1\lambda_1)\bar{v}(\vec{k}_2\lambda_2)(n\gamma)v(\vec{p}_2\sigma_2)B^{(0)}(nM|\tilde{p}_1\tilde{p}_2; \tilde{k}_1\tilde{k}_2) \right]. \end{aligned} \quad (8)$$

Scalar functions $A^{(0)}$ and $B^{(0)}$ describing the properties of quark-quark interactions in QCD in the given approximation are defined with the help of

the following integrals

$$\begin{aligned}
A^{(0)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= \\
&= \frac{g^2}{(2\pi)^5} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\
&\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \frac{1}{[p - k - (\alpha - \beta)n]^2 + i\varepsilon}, \quad (9)
\end{aligned}$$

$$\begin{aligned}
B^{(0)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= \\
&= \frac{g^2}{(2\pi)^5} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\
&\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \frac{(\alpha - \beta)^2 - (\Delta - \Delta')^2/4}{([p - k - (\alpha - \beta)n]^2 + i\varepsilon)^2}, \quad (10)
\end{aligned}$$

The integrals defining the functions $A^{(0)}$ and $B^{(0)}$ may explicitly be calculated. Here we shall present the calculation results for the special evolution gauge (Markov–Yukawa gauge), where the normal vector n is directed along the total momentum of the system, and for the case when the quark and antiquark masses are equal: $m_1 = m_2 = m$, $p_{1\perp} = -p_{2\perp} = p_\perp$, $k_{1\perp} = -k_{2\perp} = k_\perp$, (in Appendix A one can find the expressions for the functions $A^{(0)}$ and $B^{(0)}$ in an arbitrary gauge and when quark and antiquark masses are not equal),

$$\begin{aligned}
A^{(0)}(M \mid p_\perp; k_\perp) &= \quad (11) \\
&= \frac{g^2}{(2\pi)^3} \cdot \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)},
\end{aligned}$$

$$\begin{aligned}
B^{(0)}(M \mid p_\perp; k_\perp) &= \quad (12) \\
&= \frac{g^2}{2(2\pi)^3} [(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} - M) \times \\
&\times \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^2} + \\
&+ \frac{(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{-(p_\perp - k_\perp)^2(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^2} +
\end{aligned}$$

$$+ \frac{(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{(\sqrt{-(p_\perp - k_\perp)^2})^3(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)}],$$

The calculated functions $A^{(0)}$ and $B^{(0)}$ correspond to the one-gluon exchange approximation. Note that the function $A^{(0)}$ on the energy shell

$$M = 2\sqrt{m^2 - p_\perp^2} = 2\sqrt{m^2 - k_\perp^2}$$

takes the following form

$$A^{(0)}(M | p_\perp; k_\perp) |_{on \ shell} = \frac{g^2}{(2\pi)^3} \cdot \frac{1}{-(p_\perp - k_\perp)^2}.$$

The function $B^{(0)}$ turns into zero on the energy shell.

4 Account of Infrared Singularities of Gluon Green's Functions in QCD

Nowadays there are many works, where one can find well grounded arguments in favour of the singular infrared behaviour $M^2/(k^2)^2$ for gluon Green's functions in QCD (see, for instance, review [12] and references in it). In particular it is well known, that the linear growth of the potential of quark-antiquark interaction, which agrees with the experimental data on quarkonium spectroscopy, corresponds to the static limit of the diagram for one dressed gluon exchange, where the propagator has the mentioned infrared asymptotics. Namely this correspondence was in essence the very first and main argument in favour of the assumption on such singular infrared behaviour of the total one-particle gluon Green's function. Further studies of the QCD structure allowed one to make an important conclusion that infrared asymptotic M^2/k^4 of the gluon propagator yields a self-consistent description of the QCD infrared region. This result and success of the potential model in describing heavy quarkonium spectroscopy with the quark interaction potential in the form of a sum of a Coulomb-like term and the one linearly growing with distance, make us think that we will obtain a sufficiently good approximation for the total one-particle gluon Green's function if we present it in the following form

$$D_{\mu\nu} = D_{\mu\nu}^{(0)}(k) + D_{\mu\nu}^{(1)}(k), \quad (13)$$

where $D_{\mu\nu}^{(0)}(k)$ determines the ultraviolet behaviour of the gluon propagator and coincides with the free gluon Green's function, and $D_{\mu\nu}^{(1)}(k)$ describes the singular infrared asymptotics mentioned above so that

$$\begin{aligned} D_{\mu\nu}(k) &= D_{\mu\nu}^{(0)}(k), & k^2 \rightarrow \infty, \\ D_{\mu\nu}(k) &= D_{\mu\nu}^{(1)}(k), & k^2 \rightarrow 0. \end{aligned}$$

In the present Section we will show in the framework of the single-time reduction method, what changes happen to the two-quark interaction quasipotential in QCD if the infrared singularities of the gluon propagator are taken into consideration. A more accurate approximation we are going to make here, consists in the fact that we intend to use in (6) a total one-particle gluon Green's function instead of a free one. For the total Green's function we shall use the representation (13), where $D_{\mu\nu}^{(0)}$ is defined above with Eq.(7). And for $D_{\mu\nu}^{(1)}$ we take the representation from [12], which follows from the investigations of infrared structure of QCD,

$$D_{\mu\nu}^{(1)}(q) = \frac{\kappa^2}{(q^2 + i\varepsilon)^2} (g_{\mu\nu} + (d^{(1)} - 1) \frac{q_\mu q_\nu}{q^2 + i\varepsilon}), \quad (14)$$

where $d^{(1)}$ is a parameter, which in general does not coincide with $d^{(0)}$. In the literature on infrared problem in QCD we can find the ideas to consider the parameters $d^{(0)}$ and $d^{(1)}$ different in values. We shall come back to this important problem a little bit later.

Substituting the expression for $D_{\mu\nu}$ from (13) into the R.H.S. of (6) instead of $D_{\mu\nu}^{(0)}$ and taking into account (7) and (14), for the quark-antiquark interaction quasipotential we obtain

$$\begin{aligned} V_{q_1 \bar{q}_2}(nM \mid \tilde{p}_1 \sigma_{i_1} \tilde{p}_2 \sigma_{i_2}; \tilde{k}_2 \lambda_{j_2} \tilde{k}_1 \lambda_{j_1}) &= n_0 \delta^3(\vec{P}_M - \vec{K}_M) \delta_{\sigma_{f_1}}^{\lambda_{f_1}} \delta_{\sigma_{f_2}}^{\lambda_{f_2}} \sum_a [t_{(1)}^a]_{\sigma_{c_1}}^{\lambda_{c_1}} [t_{(2)}^a]_{\lambda_{c_2}}^{\sigma_{c_2}} \\ &\times [\bar{u}(\vec{p}_1 \sigma_1) \gamma^\mu u(\vec{k}_1 \lambda_1) \bar{v}(\vec{k}_2 \lambda_2) \gamma_\mu v(\vec{p}_2 \sigma_2) A(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) + \\ &+ \bar{u}(\vec{p}_1 \sigma_1) (n\gamma) u(\vec{k}_1 \lambda_1) \bar{v}(\vec{k}_2 \lambda_2) (n\gamma) v(\vec{p}_2 \sigma_2) B(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2)], \end{aligned} \quad (15)$$

where

$$\begin{aligned} A(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= \\ &= A^{(0)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) + A^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2), \end{aligned} \quad (16)$$

$$\begin{aligned}
B(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= (d^{(0)} - 1) B^{(0)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) + \\
&+ (d^{(1)} - 1) B^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2), \quad (17)
\end{aligned}$$

Scalar functions $A^{(1)}$ and $B^{(1)}$ are defined with the help of the following integrals [8]

$$\begin{aligned}
A^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= \\
&= \frac{-(g\kappa)^2}{(2\pi)^5} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\
&\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \frac{1}{([p - k - (\alpha - \beta)n]^2 + i\varepsilon)^2}, \quad (18)
\end{aligned}$$

$$\begin{aligned}
B^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &= \\
&= \frac{-(g\kappa)^2}{(2\pi)^5} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\
&\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \frac{(\alpha - \beta)^2 - (\Delta - \Delta')^2/4}{([p - k - (\alpha - \beta)n]^2 + i\varepsilon)^3}. \quad (19)
\end{aligned}$$

The integrals defining the functions $A^{(1)}$ and $B^{(1)}$ may explicitly be calculated. Here we shall present as in previous Section the calculation results for the special Markov–Yukawa evolution gauge and for the case when the quark and antiquark masses are equal: $m_1 = m_2 = m$, $p_{1\perp} = -p_{2\perp} = p_\perp$, $k_{1\perp} = -k_{2\perp} = k_\perp$, (in Appendix A one can find the expressions for the functions $A^{(1)}$ and $B^{(1)}$ in general case),

$$\begin{aligned}
A^{(1)}(M \mid p_\perp; k_\perp) &= \quad (20) \\
&= \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{-2(p_\perp - k_\perp)^2} \times \\
&\times \left[\frac{1}{(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^2} + \right. \\
&+ \left. \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})} \right],
\end{aligned}$$

$$\begin{aligned}
B^{(1)}(M | p_{\perp}; k_{\perp}) &= \\
&= -\frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{4(\sqrt{-(p_{\perp} - k_{\perp})^2})^3} \times \\
&\times \left[\frac{(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} - M)^2}{(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} + \sqrt{-(p_{\perp} - k_{\perp})^2} - M)^3} - \right. \\
&- \frac{3}{2} \cdot \frac{\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} - M}{(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} + \sqrt{-(p_{\perp} - k_{\perp})^2} - M)^2} - \\
&- \frac{(\sqrt{m^2 - p_{\perp}^2} - \sqrt{m^2 - k_{\perp}^2})^2}{(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} + \sqrt{-(p_{\perp} - k_{\perp})^2} - M)^3} - \\
&- \frac{3(\sqrt{m^2 - p_{\perp}^2} - \sqrt{m^2 - k_{\perp}^2})^2}{2\sqrt{-(p_{\perp} - k_{\perp})^2}(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} + \sqrt{-(p_{\perp} - k_{\perp})^2} - M)^2} - \\
&- \left. \frac{3(\sqrt{m^2 - p_{\perp}^2} - \sqrt{m^2 - k_{\perp}^2})^2}{2(\sqrt{-(p_{\perp} - k_{\perp})^2})^2(\sqrt{m^2 - p_{\perp}^2} + \sqrt{m^2 - k_{\perp}^2} + \sqrt{-(p_{\perp} - k_{\perp})^2} - M)} \right].
\end{aligned} \tag{21}$$

The functions $A^{(1)}$ and $B^{(1)}$ originated from the infrared singular part of the total gluon propagator. Note that the function $A^{(1)}$ on the energy shell $M = 2\sqrt{m^2 - p_{\perp}^2} = 2\sqrt{m^2 - k_{\perp}^2}$ takes the following form

$$A^{(1)}(M | p_{\perp}; k_{\perp})|_{on \ shell} = \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{(p_{\perp} - k_{\perp})^4}.$$

Similar to the function $B^{(0)}$ the function $B^{(1)}$ turns into zero on the energy shell.

5 Configuration Space and Local Approximations

Dynamic functions A and B , defined with formulae (16,17), characterize the interaction of two quarks in QCD in one-gluon exchange approximation with

an account of the infrared singularities of the total gluon propagator. In order to analyze the dynamic functions in the configuration space it will be more convenient to go over to new variables in these functions which are defined in the following way

$$\tilde{p}_i = L(n)\overset{\circ}{p}_i, \quad \tilde{k}_i = L(n)\overset{\circ}{k}_i,$$

where $L(n)$ is the matrix of a pure Lorentz transformation with elements

$$L(n)_0^\mu = L(n)_\mu^0 = n^\mu, \quad L(n)_j^i = \delta_j^i - (1 + n_0)^{-1} n^i n_j.$$

As is easily seen, in this case the variables defined above are transformed to the form

$$\begin{aligned} \overset{\circ}{p}_{1\perp} = -\overset{\circ}{p}_{2\perp} = \overset{\circ}{p}_\perp = (0, \overset{\circ}{p}), \quad \overset{\circ}{k}_{1\perp} = -\overset{\circ}{k}_{2\perp} = \overset{\circ}{k}_\perp = (0, \overset{\circ}{k}), \\ \overset{\circ}{p} = \overset{\circ}{p}_1 = -\overset{\circ}{p}_2, \quad \overset{\circ}{k} = \overset{\circ}{k}_1 = -\overset{\circ}{k}_2, \end{aligned} \quad (22)$$

in this,

$$p_\perp^2 = (\overset{\circ}{p}_\perp)^2 = -(\overset{\circ}{p})^2, \quad k_\perp^2 = (\overset{\circ}{k}_\perp)^2 = -(\overset{\circ}{k})^2. \quad (23)$$

In the terms of new variables and account of transformation properties of bispinors we obtain for the spinor structure of the interaction potential the following expression

$$\begin{aligned} u^+(\overset{\circ}{p}\sigma_1)u(\overset{\circ}{k}\lambda_1)v^+(-\overset{\circ}{k}\lambda_2)v(-\overset{\circ}{p}\sigma_2)[A(M | \overset{\circ}{p}; \overset{\circ}{k}) + B(M | \overset{\circ}{p}; \overset{\circ}{k})] - \\ - \bar{u}(\overset{\circ}{p}\sigma_1)\bar{\gamma}u(\overset{\circ}{k}\lambda_1)\bar{v}(-\overset{\circ}{k}\lambda_2)\bar{\gamma}v(-\overset{\circ}{p}\sigma_2)A(M | \overset{\circ}{p}; \overset{\circ}{k}), \end{aligned} \quad (24)$$

where the expressions for the functions A and B in the terms of new variables are simply derived from the relevant formulae (11,12), (20,21) through trivial substitutions (22,23). From explicit expressions (11,12,20,,21) it is seen that the dynamic functions A and B , defining the properties of the interaction potential for quark and antiquark, are non-local functions depending on the total energy of the quark-antiquark system. This result is a consequence of a consistent relativistic consideration of the two body problem in the framework of the local quantum field theory. For the quark-antiquark

configuration, where the conditions $\vec{p}^2/m^2 \ll 1$, $\vec{k}^2/m^2 \ll 1$ are fulfilled, one can approximate the dynamic functions A and B by the local functions with good accuracy. For instance for the functions $A^{(1)}$ and $B^{(1)}$ we find that

$$\begin{aligned} A^{(1)}(M | p_{\perp}; k_{\perp}) &\cong A^{(1)}(\varepsilon; | \vec{p} - \vec{k} |) = \\ &= \frac{(g\kappa)^2}{2(2\pi)^3 | \vec{p} - \vec{k} |^2} \left[\frac{1}{(| \vec{p} - \vec{k} | - \varepsilon)^2} + \frac{1}{| \vec{p} - \vec{k} | (| \vec{p} - \vec{k} | - \varepsilon)} \right], \end{aligned} \quad (25)$$

$$\begin{aligned} B^{(1)}(M | p_{\perp}; k_{\perp}) &\cong B^{(1)}(\varepsilon; | \vec{p} - \vec{k} |) = \\ &= \frac{-(g\kappa)^2}{4(2\pi)^3 | \vec{p} - \vec{k} |^3} \left[\frac{\varepsilon^2}{(| \vec{p} - \vec{k} | - \varepsilon)^3} + \frac{3}{2} \cdot \frac{\varepsilon}{(| \vec{p} - \vec{k} | - \varepsilon)^2} \right], \end{aligned} \quad (26)$$

where $\varepsilon = M - 2m$ is the binding energy of the quark–antiquark system. We obtain also corresponding local approximations for the functions $A^{(0)}$ and $B^{(0)}$ [10]

$$A^{(0)}(\varepsilon; | \vec{p} - \vec{k} |) = \frac{g^2}{(2\pi)^3} \cdot \frac{1}{| \vec{p} - \vec{k} | (| \vec{p} - \vec{k} | - \varepsilon)}, \quad (27)$$

$$B^{(0)}(\varepsilon; | \vec{p} - \vec{k} |) = \frac{g^2}{(2\pi)^3} \cdot \frac{-\varepsilon}{2 | \vec{p} - \vec{k} | (| \vec{p} - \vec{k} | - \varepsilon)^2}. \quad (28)$$

In the configuration space, to which we pass through the Fourier transformation

$$A(\varepsilon; r) = \int d\vec{q} \exp(i\vec{q}\vec{x}) A(\varepsilon; | \vec{q} |), \quad r \equiv | \vec{x} | \quad (29)$$

(and a similar integral for the function B), local energy–dependent potentials will correspond to the functions (25–28). The expressions for the functions $A^{(0)}$ and $B^{(0)}$ have the form [10]

$$A^{(0)}(\varepsilon; r) = \frac{g^2}{4\pi r} \cdot \frac{2}{\pi} \cdot a(\bar{\varepsilon}r), \quad B^{(0)}(\varepsilon; r) = \frac{g^2}{4\pi r} \cdot \frac{\bar{\varepsilon}r}{\pi} \cdot b(\bar{\varepsilon}r)$$

in the case of a negative binding energy $\varepsilon = -\bar{\varepsilon} < 0, \bar{\varepsilon} > 0$ and

$$A^{(0)}(\varepsilon; r) = \frac{g^2}{4\pi r} \cdot \frac{2}{\pi} [\pi e^{i\varepsilon r} - a(\varepsilon r)],$$

$$B^{(0)}(\varepsilon; r) = -\frac{g^2}{4\pi r} \cdot \frac{\varepsilon r}{\pi} \cdot [b(\varepsilon r) + i\pi e^{i\varepsilon r}]$$

in the case of positive binding energy $\varepsilon > 0$, where

$$a(x) = ci(x)\sin(x) - si(x)\cos(x), \quad b(x) = -ci(x)\cos(x) - si(x)\sin(x),$$

$ci(x)$ and $si(x)$ are integral cosines and sines.

As can easily be seen, integral (29), determining the functions $A^{(1)}$ and $B^{(1)}$ in the configuration space, diverge. With the help of the standard regularization procedures the divergent part in the integral for the functions $A^{(1)}$ and $B^{(1)}$ may easily be singled out. As a result we obtain

$$A^{(1)}(\varepsilon; r) = -\frac{\kappa^2}{2\bar{\varepsilon}^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{2}{\pi} \cdot \bar{\varepsilon} r [b(\bar{\varepsilon} r) - b(\mu r)], \quad \mu \rightarrow 0 \quad (30)$$

$$B^{(1)}(\varepsilon; r) = \frac{\kappa^2}{4\bar{\varepsilon}^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{\bar{\varepsilon} r}{\pi} [\bar{\varepsilon} r a(\bar{\varepsilon} r) - 1 - b(\bar{\varepsilon} r) + b(\mu r)], \quad \mu \rightarrow 0 \quad (31)$$

in the case of a negative binding energy and

$$A^{(1)}(\varepsilon; r) = \frac{\kappa^2}{2\varepsilon^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{2}{\pi} \cdot \varepsilon r [b(\varepsilon r) + i\pi e^{i\varepsilon r} - b(\mu r)], \quad \mu \rightarrow 0 \quad (32)$$

$$B^{(1)}(\varepsilon; r) = \frac{\kappa^2}{4\varepsilon^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{\varepsilon r}{\pi} [(i + \varepsilon r)\pi e^{i\varepsilon r} - \varepsilon r a(\varepsilon r) + 1 + b(\varepsilon r) - b(\mu r)], \quad \mu \rightarrow 0 \quad (33)$$

in the case of a positive binding energy. The function $b(x)$ has a logarithmic singularity at zero and, as can easily be seen, one and the same infinite constant $b(0)$ is present in expressions (30—33). Here we find one very important circumstance, which consists in the following. From the spinor structure of (24) it follows that the spin-independent part of the interaction quasipotential is determined with a linear combination of the dynamic functions $A + B$, which will be presented in the form

$$A + B = V^{(0)} + V^{(1)} \equiv V,$$

where

$$V^{(0)} = A^{(0)} + (d^{(0)} - 1)B^{(0)}, \quad (34)$$

$$V^{(1)} = A^{(1)} + (d^{(1)} - 1)B^{(1)}. \quad (35)$$

It turns out that there exists a special gauge $d^{(1)} = -3$, where the infinities mentioned above are canceled, and we come to the finite result for the function $V^{(1)}$

$$V_A^{(1)}(\varepsilon; r) \equiv V^{(1)}(\varepsilon; r) |_{d^{(1)}=-3} = \frac{\kappa^2}{\varepsilon^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{\bar{\varepsilon} r}{\pi} [1 - \bar{\varepsilon} r a(\bar{\varepsilon} r)], \quad (36)$$

in the case of the negative binding energy and in the case of this energy being positive we have

$$V_A^{(1)}(\varepsilon; r) = -\frac{\kappa^2}{\varepsilon^2} \cdot \frac{g^2}{4\pi r} \cdot \frac{\varepsilon r}{\pi} [1 - \varepsilon r a(\varepsilon r) + \pi \varepsilon r e^{i\varepsilon r}]. \quad (37)$$

This remarkable result of the cancellation of divergences, which leads to the finite function $V^{(1)}$, seems to be connected with the property of gauge $d^{(1)} = -3$ discussed in [12], which manifests itself in the fact, that in this gauge gluon Green's function is transverse in the coordinate space, which, in its turn, guarantees the existence of the static color charge field. We shall present also the expression for the function $V_A^{(1)}$ in the momentum space

$$\begin{aligned} V_A^{(1)}(M | p_\perp; k_\perp) &= \quad (38) \\ &= \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}} \times \\ &\times \left[\frac{1}{(\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^3} - \right. \\ &- \frac{(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{(\sqrt{-(p_\perp - k_\perp)^2})^2 (\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^3} - \\ &- \frac{3(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{2(\sqrt{-(p_\perp - k_\perp)^2})^3 (\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^2} - \\ &\left. - \frac{3(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{2(\sqrt{-(p_\perp - k_\perp)^2})^4 (\sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)} \right]. \end{aligned}$$

The corresponding local approximation for the function $V_A^{(1)}$, analogous to formulae (25–28), has the form

$$V_A^{(1)}(\varepsilon; |\vec{p} - \vec{k}|) = \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{|\vec{p} - \vec{k}| (|\vec{p} - \vec{k}| - \varepsilon)^3}. \quad (39)$$

One may also get convinced that the expressions in the R.H.S. of formulae (36) and (37) can be obtained through the Fourier transformation of function (39).

It will be interesting to study asymptotic properties of the function $V_A^{(1)}$ in the region of large and small distances. Using the known asymptotic expansions for $ci(x)$ and $si(x)$ [13], we obtain from (36) and (37);

a) the binding energy is negative $\varepsilon = -\bar{\varepsilon} < 0$, $\bar{\varepsilon} > 0$:

$$V_A^{(1)}(\varepsilon; r) = \frac{\kappa^2}{\bar{\varepsilon}^2} \cdot \frac{2\alpha\bar{\varepsilon}}{\pi(\bar{\varepsilon}r)^2} \left[1 - \frac{12}{(\bar{\varepsilon}r)^2} + O\left(\frac{1}{(\bar{\varepsilon}r)^4}\right) \right], \quad r \gg \frac{1}{\bar{\varepsilon}}, \quad (40)$$

$$V_A^{(1)}(\varepsilon; r) = \frac{\kappa^2}{\bar{\varepsilon}^2} \cdot \frac{\alpha\bar{\varepsilon}}{\pi} \left[1 - \frac{\pi}{2}\bar{\varepsilon}r - (\bar{\varepsilon}r)^2 (\ln(\gamma\bar{\varepsilon}r) - 1) + O((\bar{\varepsilon}r)^3) \right], \quad r \ll \frac{1}{\bar{\varepsilon}}, \quad (41)$$

b) the binding energy is positive $\varepsilon > 0$:

$$V_A^{(1)}(\varepsilon; r) = -\alpha\kappa^2 r e^{i\varepsilon r} \left[1 + O\left(\frac{1}{(\varepsilon r)^3}\right) \right], \quad r \gg \frac{1}{\varepsilon}, \quad (42)$$

$$V_A^{(1)}(\varepsilon; r) = -\frac{\kappa^2}{\varepsilon^2} \cdot \frac{\alpha\varepsilon}{\pi} \left[1 + \frac{\pi}{2}\varepsilon r - (\varepsilon r)^2 (\ln(\gamma\varepsilon r) - 1 - i\pi) + O((\varepsilon r)^3) \right], \quad r \ll \frac{1}{\varepsilon} \quad (43)$$

where we put $\alpha = g^2/4\pi$. Hence, in the discrete spectrum (the binding energy is negative) in the range of large distances the function $V_A^{(1)}$ decreases at the infinity more rapidly than the Coulomb one, which coincides with the corresponding asymptotic behaviour of the function $V^{(0)}(\varepsilon; r)$. In the region of small distances in the discrete spectrum the behaviour of the function $V_A^{(1)}$ differs greatly from the behaviour of $V^{(0)}$ which has a Coulomb singularity at zero. The function $V_A^{(1)}(\varepsilon, r)$ is inversely proportional to the binding energy with the proportionality coefficient equal to $\alpha\kappa^2/\pi$, at zero, i.e.,

$$V_A^{(1)}(\varepsilon; r) |_{r=0} = \frac{\alpha\kappa^2}{\pi\bar{\varepsilon}}, \quad \varepsilon = -\bar{\varepsilon} < 0.$$

The same difference in the behaviour of the functions $V_A^{(1)}$ and $V^{(0)}$ in the region of small distances holds in the case of a continuous spectrum, when the binding energy is positive, in this case

$$V_A^{(1)}(\varepsilon; r) |_{r=0} = -\frac{\alpha\kappa^2}{\pi\varepsilon}, \quad \varepsilon > 0.$$

Here we shall make a remark connected with the following fact. In ref.[12] it has been shown that the singular structure of the gluon propagator within the framework of the dimensional regularization used in the given paper, depends on the way of limiting transition to the physical dimensionality $n = 4$ of the space-time. In particular, it has been noted, that a self consistent description of the ghost and gluon Green's functions fixes such a transition to the physical dimensionality of the space where the singularity structure of the gluon propagator in the infrared region has the form

$$D_{\mu\nu}(k) = D_{\mu\nu}^{(1)}(k) + D_{\mu\nu}^{(2)}(k), \quad k^2 \rightarrow 0, \quad (44)$$

where

$$D_{\mu\nu}^{(2)}(k) = -2\pi^2\kappa^2 i\delta^{(4)}(k)g_{\mu\nu}. \quad (45)$$

One can easily guess that taking account of an additional term of form (45) with the help of the scheme presented above, leads to the appearance of an additional term in the function A , which will now be equal to

$$A = A^{(0)} + A^{(1)} + A^{(2)}.$$

We obtain an explicit expression for $A^{(2)}$

$$A^{(2)}(M | p_\perp; k_\perp) = -\frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{2\pi n_0 \delta^{(3)}(\vec{p}_\perp - \vec{k}_\perp)}{2\sqrt{m^2 - p_\perp^2} - M}, \quad (46)$$

which takes a very simple form in the local limit

$$A^{(2)}(\varepsilon; \overset{\circ}{\vec{p}} - \overset{\circ}{\vec{k}}) = \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{2\pi}{\varepsilon} \delta^{(3)}(\overset{\circ}{\vec{p}} - \overset{\circ}{\vec{k}}). \quad (47)$$

In the configuration space a constant distance-independent term

$$A^{(2)}(\varepsilon; r) = \frac{\alpha\kappa^2}{\pi\varepsilon} \quad (48)$$

will correspond to function (47). The function V takes an even simpler form if the additional term (48) is taken into consideration

$$\begin{aligned}
V_A(\varepsilon; r) &\equiv V_A^{(1)}(\varepsilon; r) + A^{(2)}(\varepsilon; r) = \\
&= -\frac{\alpha\kappa^2}{\pi} \cdot ra(\bar{\varepsilon}r), \quad \varepsilon = -\bar{\varepsilon} < 0, \\
&= \frac{\alpha\kappa^2}{\pi} \cdot r(a(\varepsilon r) - \pi e^{i\varepsilon r}), \quad \varepsilon > 0.
\end{aligned} \tag{49}$$

The essential difference of the changed function V_A from function $V_A^{(1)}$ manifests itself in the fact that in the expression (49) a correct transition to the limit of the zero binding energy is allowed, and as can easily be seen, only in the limit of the zero binding energy we come to the potential linearly growing with distance. Besides from formula (49) we get

$$V_A(\varepsilon; 0) = 0.$$

It is obvious that the indicated differences may essentially influence on the results of the data analysis for spectroscopy and decays of quark systems. Because of the importance of the circumstance we should also note here the changes which occur in the asymptotic behaviour of the function V_A ;

a) the binding energy is negative:

$$V_A(\varepsilon; r) = -\frac{\alpha\kappa^2}{\pi\bar{\varepsilon}} \left[1 - \frac{2}{(\bar{\varepsilon}r)^2} + O\left(\frac{1}{(\bar{\varepsilon}r)^4}\right) \right], \quad r \gg \frac{1}{\bar{\varepsilon}}, \tag{50}$$

$$V_A(\varepsilon; r) = -\frac{\alpha\kappa^2}{\pi} \cdot r \left[\frac{\pi}{2} + (\bar{\varepsilon}r)(\ln(\gamma\bar{\varepsilon}r) - 1) + O((\bar{\varepsilon}r)^2) \right], \quad r \ll \frac{1}{\bar{\varepsilon}}, \tag{51}$$

b) the binding energy is positive:

$$V_A(\varepsilon; r) = -\alpha\kappa^2 r e^{i\varepsilon r} \left[1 + O\left(\frac{1}{(\varepsilon r)}\right) \right], \quad r \gg \frac{1}{\varepsilon}, \tag{52}$$

$$V_A(\varepsilon; r) = -\frac{\alpha\kappa^2}{\pi} \cdot r \left[\frac{\pi}{2} - (\varepsilon r)(\ln(\gamma\varepsilon r) - 1 - i\pi) + O((\varepsilon r)^2) \right], \quad r \ll \frac{1}{\varepsilon}. \tag{53}$$

In the continuous spectrum in the range of large distances the function $V_A^{(1)}$ has oscillations with the amplitude, linearly growing with the distance

with the proportionality coefficient equal to $\alpha\kappa^2$. In the range of large distances in the continuous spectrum the function $V^{(0)}$ is characterized by the same oscillations, as the function $V_A^{(1)}$, but with the amplitude having two components: decreasing with the distance according to the Coulomb law and constant which proportional to the binding energy. More precisely we have

$$V^{(0)}(\varepsilon; r) = 2\alpha e^{i\varepsilon r} \left[\frac{1}{r} + \frac{d^{(0)} - 1}{2i} \cdot \varepsilon \right], r \gg \frac{1}{\varepsilon}.$$

One can see another interesting property, namely a weak dependence of the interaction quasipotential on the choice of the gauge at negative binding energy.

Contrary to the discrete spectrum in the case of the continuous spectrum (at the positive binding energy), there is quite a noticeable dependence of the interaction quasipotential $V^{(0)}$ (without taking account of the infrared singularities) on the gauge, which is characterized by the presence of the "knot" points, where the potential $V^{(0)}$ has one and the same value at any values of the gauge parameter $d^{(0)}$. Such a peculiarity in the behaviour of the interaction quasipotential is conserved even one takes account of infrared singularities, in this in the range of large distances the contribution of infrared singularities to the interaction quasipotential for quark and antiquark is decisive.

6 Asymptotic Freedom and Quark–Quark Forces in QCD

In the previous Sections the single–time reduction technique of the Bethe–Salpeter formalism for two–fermion systems [8] was applied to the problem of calculating the two–quark interaction quasipotential in the one–gluon exchange approximation in QCD. In this approximation, the analytic expressions for the quasipotential of two–quark interactions were obtained, allowing explicitly for the structure of the initial gauge model. It was shown that a consistent relativistic consideration of the quark interaction problem allows to establish a nontrivial energy dependence of the quark interaction potential. This energy dependence gives the interaction potential quite unusual properties concerning its behavior in the configuration space. In particular,

as a result of the energy dependence of the interaction potential, the character of the forces changes qualitatively during the transition from the discrete spectrum (the region of the negative values of the binding energy) to the continuous spectrum (that of the positive values of the binding energy). Namely, the smooth behavior of the interaction potential in the discrete spectrum goes into the oscillations in the continuous spectrum.

Using the ansatz about the singular behavior of a gluon propagator in the infrared region, we've explored how infrared singularities of gluon Green's functions affect the behavior of quark–quark forces in quantum chromodynamics.

The singular behavior of gluon Green's functions is a characteristic property of the non–Abelian gauge model under consideration and originates from the nonperturbative research of the infrared region in QCD [12]. Another peculiarity of quantum chromodynamics is the discovered asymptotic freedom of the model, which is testified by the decrease of a running coupling constant with the growth of a transferred momentum. This property is established by the perturbative analysis of QCD [14,15]. The asymptotic freedom allows to calculate things in perturbation theory at small distances, and to compare the results with experimentally measurable quantities at large momentum transfers or large transversal momenta.

In the present Section we will show in a consistent relativistic way how the asymptotic freedom displays the character of quark–quark forces.

6.1 Generalized Richardson's Parameterization and Single–Time Formalism in Quantum Chromodynamics

In calculations of the quasipotential of a two–quark interaction which have been presented in previous Sections for the function \tilde{R} in the R.H.S. of Eq. (5) the representation of the form (one–gluon exchange approximation)

$$\tilde{R}(p; k|K) = ig^2[t_{(1)}^a][t_{(2)}^b]\gamma_{(1)}^\mu\gamma_{(2)}^\nu D_{\mu\nu}^{ab}(p-k), \quad (54)$$

was used, where t^a are the generators of the gauge transformations, D is the gluon propagator, for which in turn the following ansatz was used:

$$D_{\mu\nu}^{ab}(q) = \delta^{ab}(D_{\mu\nu}^{(0)}(q) + D_{\mu\nu}^{(1)}(q)). \quad (55)$$

Here $D_{\mu\nu}^{(1)}$ determines the infrared behavior of the gluon propagator, and $D_{\mu\nu}^{(0)}(q)$ coincides with the free gluon Green's function. As usual, in formula (54) summation over the repeating indices is assumed. As it turned out, already the level of the one-gluon exchange approximation reveals many interesting features of the behavior of quark-quark forces. Some of these features were mentioned above. A disadvantage of representation (54) is that it does not take into account the property of asymptotic freedom discovered in quantum chromodynamics. This property, however, may easily be taken into account if one uses for the function \tilde{R} the following representation:

$$\tilde{R}(p; k|K) = i\alpha_s(q^2)[t_{(1)}^a][t_{(2)}^b]\gamma_{(1)}^\mu\gamma_{(2)}^\nu D_{\mu\nu}^{(0)ab}(q), \quad q \equiv p - k, \quad (56)$$

where $\alpha_s(q^2)$ is an invariant charge, for which we take the expression that was obtained in QCD in the one-loop approximation [14,15]

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b\alpha_s(\mu^2)\ln(Q^2/\mu^2)} + 0(\alpha_s), \quad Q^2 \equiv -q^2, \quad (57)$$

with $\alpha_s(\mu^2) \equiv g^2/4\pi \equiv \alpha_s$ a physical coupling constant, and the b parameter depending on the structure of the gauge group. For the group $SU_c(3)$ the b parameter is equal to

$$b = \frac{1}{12\pi}(33 - 2n_f),$$

with n_f the number of the quark flavors. Instead of the dimensional parameter μ^2 , it would be convenient to bring in another dimensional parameter, Λ^2 , through the relation

$$\ln\Lambda^2 = \ln\mu^2 - \frac{1}{b\alpha_s(\mu^2)}.$$

Then we have

$$\alpha_s(Q^2) = \frac{1}{b\ln(Q^2/\Lambda^2)} = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln(Q^2/\Lambda^2)}. \quad (58)$$

The applicability region of the one-loop approximation for an invariant charge is established from its derivation. This is the region of large $Q^2 : Q^2 \gg \Lambda^2$. Therefore, strictly speaking, representation (56) for the function \tilde{R} with the expression for the invariant charge in form (58) should be considered as an

asymptotic representation which works in the region of large Q^2 . Here the situation differs from (54): representation (56), although describing the ultraviolet behavior of the function \tilde{R} , is quite unfit for the description of the things in the infrared region. However, a simple trick invented by Richardson [16] is available to sew these two asymptotics. The Richardson's parameterization looks like

$$\tilde{R}(p; k|K) = i \frac{1}{b \ln(1 - q^2/\Lambda^2)} [t_{(1)}^a][t_{(2)}^a] \gamma_{(1)}^\mu \gamma_{(2)}^\nu D_{\mu\nu}^{(0)}(q), \quad q = p - k. \quad (59)$$

Then for the ultraviolet region $-q^2 \gg \Lambda^2$ we get representation (56) with the invariant charge (58), whereas for the infrared region $-q^2 \ll \Lambda^2$ we obtain the behavior which coincides in details with the behavior following from representation (54) if we put

$$g^2 \kappa^2 \equiv \Lambda^2/b.$$

Besides, a more general parameterization of the form

$$\tilde{R}(p; k|K) = i \frac{1}{b \ln(\xi - q^2/\Lambda^2)} [t_{(1)}^a][t_{(2)}^a] \gamma_{(1)}^\mu \gamma_{(2)}^\nu D_{\mu\nu}^{(0)}(q), \quad (60)$$

can be considered. Here ξ is some phenomenological parameter obeying the condition that $\xi \geq 1$. We shall call parameterization (60) the generalized Richardson's parameterization meaning that it leads to the standard parameterization (59) at $\xi = 1$. Remember, the function $D_{\mu\nu}^{(0)}$ entering in the R.H.S. of equality (60) is a free gluon propagator, for which we shall use the standard expression in the invariant gauge

$$D_{\mu\nu}^{(0)}(q) = \frac{-1}{q^2 + i\varepsilon} (g_{\mu\nu} + (d-1) \frac{q_\mu q_\nu}{q^2 + i\varepsilon}). \quad (61)$$

Our further calculations will be made with account of the generalized Richardson's parameterization and by the scheme we adhered to in previous Sections. Using representation (60) for the quark-antiquark interaction quasipotential we get

$$\begin{aligned} V_{q_1 \bar{q}_2}(nM|\tilde{p}_1 \sigma_{i_1} \tilde{p}_2 \sigma_{i_2}; \tilde{k}_2 \lambda_{j_2} \tilde{k}_1 \lambda_{j_1}) &= n_0 \delta^3(\vec{P}_M - \vec{K}_M) \delta_{\sigma_{f_1}}^{\lambda_{f_1}} \delta_{\sigma_{f_2}}^{\lambda_{f_2}} \sum_a [t_{(1)}^a]_{\sigma_{c_1}}^{\lambda_{c_1}} [t_{(2)}^a]_{\lambda_{c_2}}^{\sigma_{c_2}} \\ &\times \left[\bar{u}(\vec{p}_1 \sigma_1) \gamma^\mu u(\vec{k}_1 \lambda_1) \bar{v}(\vec{k}_2 \lambda_2) \gamma_\mu v(\vec{p}_2 \sigma_2) A(nM|\tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) + \right. \end{aligned}$$

$$+ (d-1)\bar{u}(\vec{p}_1\sigma_1)(n\gamma)u(\vec{k}_1\lambda_1)\bar{v}(\vec{k}_2\lambda_2)(n\gamma)v(\vec{p}_2\sigma_2)B(nM|\tilde{p}_1\tilde{p}_2;\tilde{k}_1\tilde{k}_2)\Big]. \quad (62)$$

The functions A and B in the R.H.S. of equality (62) are found with the help of the following integrals

$$\begin{aligned} A(nM|\tilde{p}_1\tilde{p}_2;\tilde{k}_1\tilde{k}_2) &= \\ &= \frac{1}{(2\pi)^5 b} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\ &\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) \frac{1}{[p - k - (\alpha - \beta)n]^2 + i\varepsilon} \times \\ &\times \frac{1}{\ln[\xi - ([p - k - (\alpha - \beta)n]^2 + i\varepsilon)/\Lambda^2]}, \end{aligned} \quad (63)$$

$$\begin{aligned} B(nM|\tilde{p}_1\tilde{p}_2;\tilde{k}_1\tilde{k}_2) &= \\ &= \frac{1}{(2\pi)^5 b} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \left(\frac{1}{\Delta/2 + \alpha - i\varepsilon} + \frac{1}{\Delta/2 - \alpha - i\varepsilon} \right) \times \\ &\times \left(\frac{1}{\Delta'/2 + \beta - i\varepsilon} + \frac{1}{\Delta'/2 - \beta - i\varepsilon} \right) [(\alpha - \beta)^2 - (\Delta - \Delta')^2/4] \times \\ &\times \frac{1}{([p - k - (\alpha - \beta)n]^2 + i\varepsilon)^2 \ln[\xi - ([p - k - (\alpha - \beta)n]^2 + i\varepsilon)/\Lambda^2]}. \end{aligned} \quad (64)$$

In the expressions (63) and (64) we use the same notations for the functions A and B as in formula (5). Our next Sections will be devoted to investigating these functions.

6.2 Analysis of the Function A

The integral, specifying the function A , can be conveniently transformed by introducing new variables $x = \alpha - \beta$ and $X = \frac{1}{2}(\alpha + \beta)$ instead of the integration variables α and β . One can integrate over the variable X with the help of the residue theorem, after which the expression for A reduces to the single integral

$$A = \frac{2\pi i}{(2\pi)^5 b} \cdot \frac{-1}{(\kappa_1 - \kappa_2)} \cdot I_A, \quad (65)$$

with

$$I_A = \int_{-\infty}^{\infty} dx \left(\frac{1}{\delta + x - i\varepsilon} + \frac{1}{\delta - x - i\varepsilon} \right) \times \\ \times \left(\frac{1}{\kappa_1 - x - i\varepsilon} + \frac{1}{x - \kappa_2 - i\varepsilon} \right) \frac{1}{\ln[(\kappa'_1 - x - i\varepsilon)(x - \kappa'_2 - i\varepsilon)/\Lambda^2]}. \quad (66)$$

Here the introduced notations are:

$$\kappa_{1,2} = (np - nk) \pm \sqrt{-(p - k)_\perp^2}, \quad \kappa'_{1,2} = (np - nk) \pm \sqrt{\xi\Lambda^2 - (p - k)_\perp^2}, \\ (p - k)_\perp^2 = (p - k)^2 - (np - nk)^2, \quad \delta \equiv \frac{1}{2}(\Delta + \Delta') \quad (67)$$

so that the following equalities are valid:

$$(p - k - xn)^2 + i\varepsilon = (x - \kappa_1 + i\varepsilon)(x - \kappa_2 - i\varepsilon), \\ (p - k - xn)^2 - \xi\Lambda^2 + i\varepsilon = (x - \kappa'_1 + i\varepsilon)(x - \kappa'_2 - i\varepsilon).$$

A detailed investigation of analytic structure of the integral (66) is contained in the Appendix B of this paper. But here we give the result for the integral in a particular evolution gauge where the normal vector n points along the total momentum of the system, and for the case of equal masses of the quark and antiquark: $m_1 = m_2 = m$, $np = nk = 0$. The calculation of integral (66) in arbitrary gauge in the case when the masses of quark and antiquark are not equal are available in the Appendix B. So, in the given particular evolution gauge there is:

$$A(nM|\tilde{p}_1\tilde{p}_2; \tilde{k}_1\tilde{k}_2) = A(M|p_\perp; k_\perp) = \frac{1}{(2\pi)^3 b\Lambda^2} \times \\ \times \left[\frac{1}{(q_\Lambda^2 - \delta_\Lambda^2) \ln(\xi + q_\Lambda^2 - \delta_\Lambda^2)} - \frac{1}{\ln\xi} \cdot \frac{\delta_\Lambda}{q_\Lambda(q_\Lambda^2 - \delta_\Lambda^2)} + \right. \\ \left. + \frac{\delta_\Lambda}{(\xi - 1)\sqrt{\xi - 1 + q_\Lambda^2}(\xi - 1 + q_\Lambda^2 - \delta_\Lambda^2)} + \right. \\ \left. + 2\delta_\Lambda \int_{\sqrt{\xi}}^{\infty} \frac{dy}{y} \frac{1}{[\ln^2(y^2 - \xi) + \pi^2]\sqrt{y^2 + q_\Lambda^2}(y^2 + q_\Lambda^2 - \delta_\Lambda^2)} \right], \quad (68)$$

with the following notations introduced:

$$q_\Lambda = q/\Lambda, \quad \delta_\Lambda = \delta/\Lambda, \quad q = \sqrt{-(p-k)_\perp^2},$$

$$\delta \equiv \sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} - M.$$

The R.H.S. of Eq. (68) contains the terms singular at $\xi \rightarrow 1$. However, one may easily verify that at $\xi \rightarrow 1$ the limit does exist and is equal to

$$\begin{aligned} A(M|p_\perp; k_\perp)|_{\xi=1} &\equiv A_R(M|p_\perp; k_\perp) = \frac{1}{(2\pi)^3 b \Lambda^2} \times \\ &\times \left[\frac{1}{(q_\Lambda^2 - \delta_\Lambda^2) \ln(1 + q_\Lambda^2 - \delta_\Lambda^2)} - \frac{\delta_\Lambda}{2q_\Lambda(q_\Lambda^2 - \delta_\Lambda^2)} - \frac{\delta_\Lambda}{2q_\Lambda^3(q_\Lambda^2 - \delta_\Lambda^2)} - \right. \\ &\left. - \frac{\delta_\Lambda}{q_\Lambda(q_\Lambda^2 - \delta_\Lambda^2)^2} + 2\delta_\Lambda \int_1^\infty \frac{dy}{y} \frac{1}{[\ln^2(y^2 - 1) + \pi^2] \sqrt{y^2 + q_\Lambda^2} (y^2 + q_\Lambda^2 - \delta_\Lambda^2)} \right]. \end{aligned} \quad (69)$$

Having now the expression for the function A , we can study how it behaves in the limit when $\Lambda^2 \rightarrow \infty$. Taking in the R.H.S. of Eq. (68) $\xi = e$ and tending $\Lambda^2 \rightarrow \infty$, we come to

$$A(M|p_\perp; k_\perp)|_{\xi=e, \Lambda^2 \rightarrow \infty} = \frac{1}{(2\pi)^3 b} \cdot \frac{1}{q(q + \delta)}. \quad (70)$$

The R.H.S. of (70) coincides with the previously obtained expression for the function $A^{(0)}$, provided we put $g^2 = b^{-1}$. After an analogous limiting transition in the R.H.S. of equality (69), we find that

$$A(M|p_\perp; k_\perp)|_{\xi=1, \Lambda^2 \rightarrow \infty} = \frac{\Lambda^2}{(2\pi)^3 b} \left[\frac{1}{2q^3(q + \delta)} + \frac{1}{2q^2(q + \delta)^2} \right], \quad (71)$$

which, in its turn, fully coincides with the earlier found expression for the function $A^{(1)}$ at $g^2 \kappa^2 = \Lambda^2 b^{-1}$, the latter defining the contribution from the infrared singularities of the gluon propagator. These two results – (70) and (71)– are easy to understand if one turns to the original integral (63) specifying the A function. The consideration of the limit $\Lambda^2 \rightarrow \infty$ in the R.H.S. of equality (63) at $\xi = e$ and $\xi = 1$ will obviously bring us to the integrals of the functions $A^{(0)}$ and $A^{(1)}$, respectively. (These functions were already calculated in previous Sections). Therefore, the limiting relations (70) and (71) correlate our calculations.

6.3 Analysis of the Function B

As before, the integral (64), specifying the B function, can be conveniently transformed with the help of the new integration variables $x = \alpha - \beta$ and $X = \frac{1}{2}(\alpha + \beta)$ and via the integration over the variable X . As a result, the expression for the function B turns out to be nothing but the difference of two single integrals

$$B = B^{(1)} - \frac{(\Delta - \Delta')^2}{4} B^{(2)}, \quad (72)$$

and besides,

$$B^{(1)} = \frac{2\pi i}{(2\pi)^3 b(\kappa_1 - \kappa_2)^2} I_B^{(1)}, \quad B^{(2)} = \frac{2\pi i}{(2\pi)^3 b(\kappa_1 - \kappa_2)^2} I_B^{(2)},$$

with

$$I_B^{(1)} = \int_{-\infty}^{\infty} dx \left(\frac{1}{\delta + x - i\varepsilon} + \frac{1}{\delta - x - i\varepsilon} \right) \times \\ \times \left(\frac{1}{\kappa_1 - x - i\varepsilon} + \frac{1}{x - \kappa_2 - i\varepsilon} \right)^2 \frac{x^2}{\ln[(\kappa'_1 - x - i\varepsilon)(x - \kappa'_2 - i\varepsilon)/\Lambda^2]}, \quad (73)$$

$$I_B^{(2)} = \int_{-\infty}^{\infty} dx \left(\frac{1}{\delta + x - i\varepsilon} + \frac{1}{\delta - x - i\varepsilon} \right) \times \\ \times \left(\frac{1}{\kappa_1 - x - i\varepsilon} + \frac{1}{x - \kappa_2 - i\varepsilon} \right)^2 \frac{1}{\ln[(\kappa'_1 - x - i\varepsilon)(x - \kappa'_2 - i\varepsilon)/\Lambda^2]}. \quad (74)$$

Here we use again the notations from the expression for the function A . The results below are the calculations of integrals (73) and (74) in a particular evolution gauge and for equal quark and antiquark masses. Again, as before, the complete exploration of these integrals is presented in the Appendix B. For the function $B^{(1)}$, therefore, we have

$$B^{(1)}(M|p_{\perp}; k_{\perp}) = \\ = -\frac{1}{(2\pi)^3 b\Lambda^2} \cdot \frac{\delta_{\Lambda}}{2q_{\Lambda}} \left[\frac{2\delta_{\Lambda}}{(q_{\Lambda} + \delta_{\Lambda})(q_{\Lambda}^2 - \delta_{\Lambda}^2) \ln(\xi + q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \right. \\ \left. - \frac{1}{\ln\xi} \cdot \frac{1}{(q_{\Lambda}^2 - \delta_{\Lambda}^2)} + \frac{2\sqrt{\xi - 1 + q_{\Lambda}^2}}{(\xi - 1)(\xi - 1 + q_{\Lambda}^2 - \delta_{\Lambda}^2)(q_{\Lambda} + \sqrt{\xi - 1 + q_{\Lambda}^2})} + \right.$$

$$+ \int_{\sqrt{\xi}}^{\infty} \frac{dy}{y} \frac{4\sqrt{y^2 + q_{\Lambda}^2}}{[ln^2(y^2 - \xi) + \pi^2](q_{\Lambda} + \sqrt{y^2 + q_{\Lambda}^2})(y^2 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} \Big]. \quad (75)$$

Here, again, we have the terms singular at $\xi \rightarrow 1$. It makes no difficulty to verify that, as before, the limit at $\xi \rightarrow 1$ does exist and looks like

$$\begin{aligned} B^{(1)}(M|p_{\perp}; k_{\perp})|_{\xi=1} \equiv B_R^{(1)}(M|p_{\perp}; k_{\perp}) &= \frac{1}{(2\pi)^3 b \Lambda^2} \cdot \frac{\delta_{\Lambda}}{2q_{\Lambda}} \left[\frac{1}{2(q_{\Lambda}^2 - \delta_{\Lambda}^2)} + \right. \\ &+ \frac{1}{(q_{\Lambda}^2 - \delta_{\Lambda}^2)^2} - \frac{1}{4q_{\Lambda}^2(q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \frac{2\delta_{\Lambda}}{(q_{\Lambda} + \delta_{\Lambda})(q_{\Lambda}^2 - \delta_{\Lambda}^2)ln(1 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \\ &\left. - \int_1^{\infty} \frac{dy}{y} \frac{4\sqrt{y^2 + q_{\Lambda}^2}}{[ln^2(y^2 - 1) + \pi^2](q_{\Lambda} + \sqrt{y^2 + q_{\Lambda}^2})(y^2 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} \right]. \quad (76) \end{aligned}$$

Calculating the integral for the function $B^{(2)}$ (in the above- mentioned particular gauge), we are brought to :

$$\begin{aligned} B^{(2)}(M|p_{\perp}; k_{\perp}) &= -\frac{1}{(2\pi)^3 b \Lambda^4} \cdot \frac{1}{2q_{\Lambda}} \times \\ &\times \left[\frac{2}{(q_{\Lambda} + \delta_{\Lambda})(q_{\Lambda}^2 - \delta_{\Lambda}^2)ln(\xi + q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \frac{1}{ln\xi} \cdot \frac{\delta_{\Lambda}}{q_{\Lambda}^2(q_{\Lambda}^2 - \delta_{\Lambda}^2)} + \right. \\ &+ \frac{2\delta_{\Lambda}}{(\xi - 1)\sqrt{\xi - 1 + q_{\Lambda}^2}(\xi - 1 + q_{\Lambda}^2 - \delta_{\Lambda}^2)(q_{\Lambda} + \sqrt{\xi - 1 + q_{\Lambda}^2})} + \\ &\left. + \int_{\sqrt{\xi}}^{\infty} \frac{dy}{y} \frac{4\delta_{\Lambda}}{[ln^2(y^2 - \xi) + \pi^2]\sqrt{y^2 + q_{\Lambda}^2}(q_{\Lambda} + \sqrt{y^2 + q_{\Lambda}^2})(y^2 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} \right]. \quad (77) \end{aligned}$$

After the limiting transition $\xi \rightarrow 1$ in the R.H.S. of equality (77), the result will be

$$\begin{aligned} B^{(2)}(M|p_{\perp}; k_{\perp})|_{\xi=1} \equiv B_R^{(2)}(M|p_{\perp}; k_{\perp}) &= \frac{1}{(2\pi)^3 b \Lambda^4} \cdot \frac{1}{2q_{\Lambda}} \left[\frac{\delta_{\Lambda}}{q_{\Lambda}^2(q_{\Lambda}^2 - \delta_{\Lambda}^2)^2} + \right. \\ &+ \frac{\delta_{\Lambda}}{2q_{\Lambda}^2(q_{\Lambda}^2 - \delta_{\Lambda}^2)} + \frac{3\delta_{\Lambda}}{4q_{\Lambda}^4(q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \frac{2}{(q_{\Lambda} + \delta_{\Lambda})(q_{\Lambda}^2 - \delta_{\Lambda}^2)ln(1 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} - \\ &\left. - \int_1^{\infty} \frac{dy}{y} \frac{4\delta_{\Lambda}}{[ln^2(y^2 - 1) + \pi^2]\sqrt{y^2 + q_{\Lambda}^2}(q_{\Lambda} + \sqrt{y^2 + q_{\Lambda}^2})(y^2 + q_{\Lambda}^2 - \delta_{\Lambda}^2)} \right]. \quad (78) \end{aligned}$$

Expressions (72), (75), (77) describe the analytic structure of the function B . It would also be useful to look at the behavior of this function in the limit $\Lambda^2 \rightarrow \infty$. Assuming for the R.H.S. of (75) $\xi = e$ and tending $\Lambda^2 \rightarrow \infty$, for the function $B^{(1)}$ we get

$$B^{(1)}(M|p_\perp; k_\perp)|_{\xi=e, \Lambda^2 \rightarrow \infty} = \frac{1}{(2\pi)^3 b} \cdot \frac{\delta}{2q(q+\delta)^2}. \quad (79)$$

After performing an analogous procedure in the R.H.S. of (77), for the function $B^{(2)}$ we find that

$$B^{(2)}(M|p_\perp; k_\perp)|_{\xi=e, \Lambda^2 \rightarrow \infty} = -\frac{1}{(2\pi)^3 b} \cdot \frac{1}{2q} \left[\frac{1}{q^2(q+\delta)} + \frac{1}{q(q+\delta)^2} \right], \quad (80)$$

and, hence, the complete function B in this limiting case becomes

$$B(M|p_\perp; k_\perp)|_{\xi=e, \Lambda^2 \rightarrow \infty} = \frac{1}{(2\pi)^3 b} \left[\frac{\delta}{2q(q+\delta)^2} + \frac{(\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2}{2q} \left(\frac{1}{q^2(q+\delta)} + \frac{1}{q(q+\delta)^2} \right) \right]. \quad (81)$$

Remember, here $\delta = \sqrt{m^2 - p_\perp^2} + \sqrt{m^2 - k_\perp^2} - M$, and besides, we take into account that $\frac{1}{4}(\Delta - \Delta')^2 = (\sqrt{m^2 - p_\perp^2} - \sqrt{m^2 - k_\perp^2})^2$. Expression (81) coincides exactly with the expression for the function $B^{(0)}$ that was obtained before under the condition that $g^2 = b^{-1}$.

The limiting transition $\Lambda^2 \rightarrow \infty$ in the R.H.S. of Eq. (76) for the function $B^{(1)}$ leads to

$$B^{(1)}(M|p_\perp; k_\perp)|_{\xi=1, \Lambda^2 \rightarrow \infty} = \frac{\Lambda^2}{(2\pi)^3 b} \cdot \frac{1}{4q} \left[\frac{3\delta}{2q^2(q+\delta)^2} - \frac{\delta^2}{q^2(q+\delta)^3} \right]. \quad (82)$$

Repeating the same limiting procedure in the R.H.S. of Eq. (84), for the function $B^{(2)}$ we obtain that

$$B^{(2)}(M|p_\perp; k_\perp)|_{\xi=1, \Lambda^2 \rightarrow \infty} = -\frac{\Lambda^2}{(2\pi)^3 b} \cdot \frac{1}{4q} \times \left[\frac{1}{q^2(q+\delta)^3} + \frac{3}{2q^3(q+\delta)^2} + \frac{3}{2q^4(q+\delta)} \right]. \quad (83)$$

Thus, in the given limiting case for the complete function B we have

$$B(M|p_{\perp}; k_{\perp})|_{\xi=1, \Lambda^2 \rightarrow \infty} = \frac{\Lambda^2}{(2\pi)^3 b} \cdot \frac{1}{4q^3} \left[\frac{3\delta}{2(q+\delta)^2} - \frac{\delta^2}{(q+\delta)^3} + \right. \\ \left. + (\sqrt{m^2 - p_{\perp}^2} - \sqrt{m^2 - k_{\perp}^2})^2 \left(\frac{1}{(q+\delta)^3} + \frac{3}{2q(q+\delta)^2} + \frac{3}{2q^2(q+\delta)} \right) \right]. \quad (84)$$

The resulting expression (84) for the function B coincides with the expression for the function $B^{(1)}$ from Eq. (21) at $g^2 \kappa^2 = \Lambda^2 b^{-1}$. Looking back at the original integrals, (73) and (74), specifying the function B , we see that the results (81) and (84), similar to those of the previous Section, have not come unexpected but rather as a correlation of our calculations. Further, it would be convenient to explore the properties of the dynamic functions, A and B , going over to the configuration space; that is what we'll do in the next Section.

6.4 Analysis of the Dynamic Functions in Configuration Space

To analyze the dynamic functions A and B in the configuration space, pass over in these functions to new variables defined as

$$\tilde{p}_i = L(n) \overset{\circ}{p}_i, \quad \tilde{k}_i = L(n) \overset{\circ}{k}_i,$$

with $L(n)$ the matrix of the Lorentz boost which has the property

$$L^{-1}(n)n = (1, \vec{0}).$$

It is easy to see that in the given particular gauge the new momentum variables are as follows:

$$\overset{\circ}{p}_{1\perp} = -\overset{\circ}{p}_{2\perp} = \overset{\circ}{p}_{\perp} = (0, \vec{p}), \quad \overset{\circ}{k}_{1\perp} = -\overset{\circ}{k}_{2\perp} = \overset{\circ}{k}_{\perp} = (0, \vec{k}), \\ \overset{\circ}{p} = \overset{\circ}{p}_1 = -\overset{\circ}{p}_2, \quad \overset{\circ}{k} = \overset{\circ}{k}_1 = -\overset{\circ}{k}_2.$$

Besides,

$$p_{\perp}^2 = (\overset{\circ}{p}_{\perp})^2 = -(\vec{p})^2, \quad k_{\perp}^2 = (\overset{\circ}{k}_{\perp})^2 = -(\vec{k})^2,$$

and hence,

$$q = \sqrt{-(p-k)_\perp^2} = \sqrt{(\vec{p} - \vec{k})^2} = |\vec{p} - \vec{k}|,$$

$$\delta = \sqrt{m^2 + \vec{p}^2} + \sqrt{m^2 + \vec{k}^2} - M.$$

The dynamic functions A and B , as their explicit expressions show, are non-local functions depending on the spectral parameter M , the latter taking the values of the total energy of a two-fermion system. This result follows from a thorough consideration of the problem of a relativistic particle interaction in the framework of the local quantum field theory. Pay attention to that the whole nonlocality of the dynamic functions concentrates in the quantity δ contained therein. The δ determines the off energy shell continuation symmetric in the particle momenta of the initial and final states. On energy shell, δ and, consequently, the B functions turn into zero. As for the A function, on energy shell it becomes local and coincides with the known Richardson's potential.

As we already know, to achieve the locality of the dynamic functions by restricting them to the energy shell means rather a destructive trick, since in this case many of the dynamic properties of a relativistic interaction are lost. Another way to locally approximate the dynamic functions is to keep to only such configurations of interacting particles for which the conditions $\vec{p}^2/m^2 \ll 1, \vec{k}^2/m^2 \ll 1$ are true. Then the quantity δ is supposed to be equal to the defect of the mass of the system or, which is the same, to the binding energy with the opposite sign, whereas the dynamic functions determining the potential become local functions with the dependence on the binding energy of the system. After such a procedure, for the function A_R , for instance, the following local approximation will be obtained:

$$A_R(M|p_\perp; k_\perp) \cong A_R(\varepsilon, \Lambda; q) =$$

$$= \frac{G}{\Lambda^2} \left[\frac{1}{[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2] \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} + \right.$$

$$+ \frac{\varepsilon_\Lambda}{2q_\Lambda[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} + \frac{\varepsilon_\Lambda}{2q_\Lambda^3[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} + \frac{\varepsilon_\Lambda}{q_\Lambda[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]^2} -$$

$$\left. - 2\varepsilon_\Lambda \int_1^\infty \frac{dy}{y} \frac{1}{[ln^2(y^2 - 1) + \pi^2] \sqrt{y^2 + q_\Lambda^2} [y^2 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} \right]. \quad (85)$$

Here $G = [(2\pi)^3 b]^{-1}$, $\varepsilon_\Lambda \equiv \varepsilon/\Lambda$, $\varepsilon = M - 2m$ is the binding energy of the system, and there in the R.H.S. of Eq. (85) an explicit indication is contained by which rule to bypass the singularities in the integral, determining the transition to the configuration space.

$$\tilde{A}_R(\varepsilon, \Lambda; r) = \frac{4\pi}{r} \int_0^\infty q dq \sin(qr) A_R(\varepsilon, \Lambda; q). \quad (86)$$

Note, this prescription is, on the one hand, due to the causal structure of the local quantum field theory, which is used as the framework for the construction of the given single-time formalism, and, on the other hand, it itself guarantees the causal properties of the dynamic equations in the single-time formalism [8,17].

Expression (85) for the function A_R has five terms. Hence, in order to pass over to the configuration space, one has to calculate, respectively, five integrals of form (86). Each of the five integrals can be found in the Appendix C. Here we only present the complete result for the whole function A_R in the particular evolution gauge. As usual, we'll distinguish between the regions of positive and negative values of the binding energy and, besides, restrict ourselves to the values of the binding energy which fulfil the condition $\varepsilon_\Lambda^2 < 1$. Thus, at a positive binding energy, for the function A_R we get

$$\begin{aligned} \tilde{A}_R(\varepsilon, \Lambda; r) = & \frac{G\pi^2}{r} \left[2e^{i\varepsilon r} \left(1 + \frac{i}{\varepsilon_\Lambda} \Lambda r \right) - \frac{2}{\pi} [a(\varepsilon r) - \frac{1}{\varepsilon_\Lambda} \Lambda r b(\varepsilon r)] - \right. \\ & - \frac{\Lambda r}{\varepsilon_\Lambda} \left[i + \frac{2}{\pi} b(\mu r) \right] \Big|_{\mu \rightarrow 0} - 4 \int_1^\infty \frac{dy}{y} \frac{\exp(-\Lambda r \sqrt{y^2 - \varepsilon_\Lambda^2})}{\ln^2(y^2 - 1) + \pi^2} - \\ & \left. - \frac{8\varepsilon_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty t dt \frac{\sin(\Lambda r t)}{\sqrt{t^2 + y^2}(t^2 + y^2 - \varepsilon_\Lambda^2)} \right], \quad (87) \end{aligned}$$

In the region of negative values of the binding energy $\varepsilon \equiv -\bar{\varepsilon} < 0$, for the function A_R we find

$$\begin{aligned} \tilde{A}_R(\varepsilon, \Lambda; r) = & \frac{G\pi^2}{r} \left[\frac{2}{\pi} [a(\bar{\varepsilon} r) - \frac{1}{\bar{\varepsilon}_\Lambda} \Lambda r b(\bar{\varepsilon} r)] + \right. \\ & \left. + \frac{\Lambda r}{\bar{\varepsilon}_\Lambda} \left[i + \frac{2}{\pi} b(\mu r) \right] \Big|_{\mu \rightarrow 0} - 4 \int_1^\infty \frac{dy}{y} \frac{\exp(-\Lambda r \sqrt{y^2 - \bar{\varepsilon}_\Lambda^2})}{\ln^2(y^2 - 1) + \pi^2} + \right. \end{aligned}$$

$$+ \frac{8\bar{\varepsilon}_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty t dt \frac{\sin(\Lambda r t)}{\sqrt{t^2 + y^2}(t^2 + y^2 - \bar{\varepsilon}_\Lambda^2)} \Big]. \quad (88)$$

When calculating the integral (86) for the function A_R , we come across the divergent integral stemming from the third term in the R.H.S. of Eq. (85). We work with this integral exploiting the standard regularization procedure. This fact shows itself in the presence of an infinite constant $b(0)$ in the R.H.S.'s of Eqs. (87) and (88). Below, this fact will be further discussed.

When the local approximation for the function B_R is constructed in that way, the function $B_R^{(2)}$ will yield no contribution, since in the local limit under consideration the factor before this function, $(\sqrt{m^2 + \vec{p}^2} - \sqrt{m^2 + \vec{k}^2})^2$, turns into zero. As a result, the local approximation for the function B_R will be:

$$\begin{aligned} B_R(M|p_\perp; k_\perp) &\cong B_R^{(1)}(\varepsilon, \Lambda; q) = \\ &= \frac{G}{\Lambda^2} \cdot \frac{\varepsilon_\Lambda}{2q_\Lambda} \left[\frac{-2\varepsilon_\Lambda}{(q_\Lambda - \varepsilon_\Lambda - i0)[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2] \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} - \right. \\ &\quad \left. - \frac{1}{2[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} - \frac{1}{[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]^2} + \frac{1}{4q_\Lambda^2[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} + \right. \\ &\quad \left. + \int_1^\infty \frac{dy}{y} \frac{4\sqrt{y^2 + q_\Lambda^2}}{[\ln^2(y^2 - 1) + \pi^2](q_\Lambda + \sqrt{y^2 + q_\Lambda^2})[y^2 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} \right]. \quad (89) \end{aligned}$$

Here, again, to go over to the configuration space, one has to calculate five integrals of form (86), corresponding to the five terms entering in the R.H.S. of Eq. (89). Referring the reader, as in the previous case with the function A_R , to the Appendix C where the details of calculations can be found, here we write down the complete result for the function B_R in the configuration space in the particular evolution gauge. For positive values of the binding energy we have that

$$\begin{aligned} \tilde{B}_R(\varepsilon, \Lambda; r) &= -\frac{G\pi^2}{r} \left[e^{i\varepsilon r} \left[i\varepsilon r - \frac{i}{2\varepsilon_\Lambda} \Lambda r - \frac{1}{2}(\Lambda r)^2 - \frac{1}{3}\varepsilon_\Lambda^2 \right] + \right. \\ &\quad \left. + \frac{2}{\pi} \left[\left(\frac{3}{4\varepsilon_\Lambda^2} - \frac{1}{2} \right) a(\varepsilon r) + \frac{1}{2\varepsilon_\Lambda} \Lambda r b(\varepsilon r) \right] + \frac{\Lambda r}{4\varepsilon_\Lambda} \left[i + \frac{2}{\pi} b(\mu r) \right] \right]_{\mu \rightarrow 0} + \end{aligned}$$

$$\begin{aligned}
& + \frac{4\varepsilon_\Lambda^2}{\pi} \int_0^\infty dt \frac{\exp(-\Lambda rt) [\varepsilon_\Lambda \ln|1 - \varepsilon_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}{(t^2 + \varepsilon_\Lambda^2)^2 [ln^2|1 - \varepsilon_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]} - \\
& - \frac{8\varepsilon_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{ln^2(y^2 - 1) + \pi^2} \int_0^\infty dt \frac{\sqrt{t^2 + y^2} \sin(\Lambda rt)}{(t + \sqrt{t^2 + y^2})(t^2 + y^2 - \varepsilon_\Lambda^2)} \Big]. \quad (90)
\end{aligned}$$

At negative values of the binding energy, $\varepsilon = -\bar{\varepsilon} < 0$, $\bar{\varepsilon} > 0$, for the function B_R there is

$$\begin{aligned}
& \tilde{B}_R(\varepsilon, \Lambda; r) = \\
& = -\frac{G\pi^2}{r} \left[\frac{2}{\pi} \left[\left(\frac{1}{2} - \frac{3}{4\bar{\varepsilon}_\Lambda^2} \right) a(\bar{\varepsilon}r) - \frac{1}{2\bar{\varepsilon}_\Lambda} \Lambda r b(\bar{\varepsilon}r) \right] - \frac{\Lambda r}{4\bar{\varepsilon}_\Lambda} \left[i + \frac{2}{\pi} b(\mu r) \right] \Big|_{\mu \rightarrow 0} + \right. \\
& + \frac{4\bar{\varepsilon}_\Lambda^2}{\pi} \int_0^\infty dt \frac{\exp(-\Lambda rt) [-\bar{\varepsilon}_\Lambda \ln|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]}{(t^2 + \bar{\varepsilon}_\Lambda^2)^2 [ln^2|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]} + \\
& \left. + \frac{8\bar{\varepsilon}_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{ln^2(y^2 - 1) + \pi^2} \int_0^\infty dt \frac{\sqrt{t^2 + y^2} \sin(\Lambda rt)}{(t + \sqrt{t^2 + y^2})(t^2 + y^2 - \bar{\varepsilon}_\Lambda^2)} \right]. \quad (91)
\end{aligned}$$

In expressions (90) and (91) for the function B_R , the same infinite constant $b(0)$ occurs as in expressions (87) and (88) describing the function A_R . This infiniteness is due to the divergency of the integral arising from the integration of the fourth term in the R.H.S. of Eq. (89) when going over to the configuration space. The situation with the divergencies resembles the case when the contribution from the infrared singularities of gluon Green's functions into the two-quark interaction potential was investigated. The analogy will seem still deeper if we remember that there exists the special gauge $d = -3$, which leads to the cancellation of the divergencies in the linear combination

$$V = A + (d - 1)B,$$

determining the spin independent part of the interaction potential. So, here is the final result for the function

$$V_A = V|_{d=-3} = A - 4B.$$

Introducing an analogous linear combination:

$$V_R = A_R + (d - 1)B_R, \quad (92)$$

one can easily get convinced that in the case under consideration in the gauge $d = -3$ the above divergencies cancel out, and we get the final function

$$V_{AR} = V_R|_{d=-3} = A_R - 4B_R. \quad (93)$$

In the region of positive values of the binding energy the expression for the function V_{AR} becomes

$$\begin{aligned} V_{AR}(\varepsilon, \Lambda; r) = & \frac{G\pi^2}{r} \left\{ 2e^{i\varepsilon r} \left[1 - \frac{2}{3}\varepsilon_\Lambda^2 + 2i\varepsilon r - (\Lambda r)^2 \right] + \right. \\ & + \frac{6}{\pi} \left[\left(\frac{1}{\varepsilon_\Lambda^2} - 1 \right) a(\varepsilon r) + \frac{1}{\varepsilon_\Lambda^2} \varepsilon r b(\varepsilon r) \right] - 4 \int_1^\infty \frac{dy}{y} \frac{\exp(-\Lambda r \sqrt{y^2 - \varepsilon_\Lambda^2})}{\ln^2(y^2 - 1) + \pi^2} - \\ & - \frac{8\varepsilon_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty t dt \frac{\sin(\Lambda r t)}{\sqrt{t^2 + y^2}(t^2 + y^2 - \varepsilon_\Lambda^2)} + \\ & + \frac{16\varepsilon_\Lambda^2}{\pi} \int_0^\infty dt \frac{\exp(-\Lambda r t) [\varepsilon_\Lambda \ln|1 - \varepsilon_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}{(t^2 + \varepsilon_\Lambda^2)^2 [\ln^2|1 - \varepsilon_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]} - \\ & \left. - \frac{32\varepsilon_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty dt \frac{\sqrt{t^2 + y^2} \sin(\Lambda r t)}{(t + \sqrt{t^2 + y^2})(t^2 + y^2 - \varepsilon_\Lambda^2)} \right\}. \quad (94) \end{aligned}$$

At negative values of the binding energy for the function V_{AR} we find

$$\begin{aligned} V_{AR}(\varepsilon, \Lambda; r) = & \frac{G\pi^2}{r} \left\{ \frac{6}{\pi} \left[\left(1 - \frac{1}{\bar{\varepsilon}_\Lambda^2} \right) a(\bar{\varepsilon} r) - \frac{1}{\bar{\varepsilon}_\Lambda^2} \bar{\varepsilon} r b(\bar{\varepsilon} r) \right] - 4 \int_1^\infty \frac{dy}{y} \frac{\exp(-\Lambda r \sqrt{y^2 - \bar{\varepsilon}_\Lambda^2})}{\ln^2(y^2 - 1) + \pi^2} + \right. \\ & + \frac{8\bar{\varepsilon}_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty t dt \frac{\sin(\Lambda r t)}{\sqrt{t^2 + y^2}(t^2 + y^2 - \bar{\varepsilon}_\Lambda^2)} + \\ & + \frac{16\bar{\varepsilon}_\Lambda^2}{\pi} \int_0^\infty dt \frac{\exp(-\Lambda r t) [-\bar{\varepsilon}_\Lambda \ln|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]}{(t^2 + \bar{\varepsilon}_\Lambda^2)^2 [\ln^2|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]} + \\ & \left. + \frac{32\bar{\varepsilon}_\Lambda}{\pi} \int_1^\infty \frac{dy}{y} \frac{1}{\ln^2(y^2 - 1) + \pi^2} \int_0^\infty dt \frac{\sqrt{t^2 + y^2} \sin(\Lambda r t)}{(t + \sqrt{t^2 + y^2})(t^2 + y^2 - \bar{\varepsilon}_\Lambda^2)} \right\}. \quad (95) \end{aligned}$$

It may be proved that the known Richardson's potential is derived from the function V_{AR} in the zero binding energy limit. Passing over to the limit of zero binding energy, in any of the expressions, (94) or (95), for the function V_{AR} we get, as should really be, one and the same result of the form

$$\lim_{\varepsilon \rightarrow 0} V_{AR}(\varepsilon, \Lambda; r) = G\pi^2 \left[\frac{f(\Lambda r)}{r} - \Lambda^2 r + C \right], \quad (96)$$

where

$$f(x) \equiv 1 - 4 \int_1^\infty \frac{dt}{t} \frac{\exp(-xt)}{\ln^2(t^2 - 1) + \pi^2},$$

$$C \equiv \lim_{\varepsilon \rightarrow 0} \frac{2\Lambda^2}{\pi\varepsilon}.$$

The comparison of Eqs. (94), (95) and (96) shows that the energy dependence changes considerably the nature of the interaction: the smooth behavior of the function V_{AR} in the region of negative values of the binding energy goes into oscillations at its positive values. Moreover, we observe that in the region of large distances the function V_{AR} starts to behave essentially in a different way depending on which particular values the binding energy takes. For positive values of the binding energy formula (94) gives

$$V_{AR}(\varepsilon, \Lambda; r) = -2\pi^2 G\Lambda^2 r e^{i\varepsilon r}, \quad r \gg \frac{1}{\varepsilon} > \frac{1}{\Lambda}. \quad (97)$$

The asymptotic behavior of the function V_{AR} in the region of large distances and for negative values of the binding energy follows from expression (95) and has the form

$$V_{AR}(\varepsilon, \Lambda; r) = \frac{4\pi G\Lambda^2}{\bar{\varepsilon}} \cdot \frac{1}{(\bar{\varepsilon}r)^2} \left[1 - 4\left(1 + \frac{\bar{\varepsilon}_\Lambda^2}{\ln(1 - \bar{\varepsilon}_\Lambda^2)}\right) + \frac{3}{2}\bar{\varepsilon}_\Lambda^2 \right], \quad (98)$$

$$r \gg \frac{1}{\bar{\varepsilon}} > \frac{1}{\Lambda}.$$

Therefore, at negative values of the binding energy the function V_{AR} is characterized at large distances by a rapider than the Coulomb decrease, whereas at positive values of the binding energy there appear oscillations of the function V_{AR} , their amplitude growing linearly with distance.

In the region of small distances, $r \ll \frac{1}{\Lambda} < \frac{1}{|\varepsilon|}$, from expressions (94) and (95) for the function V_{AR} we get

a) the binding energy positive $\varepsilon > 0$:

$$V_{AR}(\varepsilon, \Lambda; r) = \frac{G\pi^2}{r} \left[c(\varepsilon_\Lambda) + \frac{1}{\ln(\frac{1}{\Lambda r} \sqrt{1 + (\varepsilon r)^2})} \right], \quad (99)$$

b) the binding energy negative $\varepsilon = -\bar{\varepsilon} < 0$:

$$V_{AR}(\varepsilon, \Lambda; r) = \frac{G\pi^2}{r} \left[\bar{c}(\bar{\varepsilon}_\Lambda) + \frac{1}{\ln(\frac{1}{\Lambda r} \sqrt{1 + (\bar{\varepsilon} r)^2})} \right], \quad (100)$$

the functions c and \bar{c} here being of the form:

$$c(\varepsilon_\Lambda) = -2 - \frac{4}{3}\varepsilon_\Lambda^2 + \frac{3}{\varepsilon_\Lambda^2} + \frac{16\varepsilon_\Lambda^2}{\pi} \int_0^\infty dt \frac{\varepsilon_\Lambda \ln|1 - \varepsilon_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \varepsilon_\Lambda^2 - 1)}{(t^2 + \varepsilon_\Lambda^2)^2 [\ln^2|1 - \varepsilon_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}, \quad (101)$$

$$\bar{c}(\bar{\varepsilon}_\Lambda) = 2 - \frac{3}{\bar{\varepsilon}_\Lambda^2} + \frac{16\bar{\varepsilon}_\Lambda^2}{\pi} \int_0^\infty dt \frac{-\bar{\varepsilon}_\Lambda \ln|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)}{(t^2 + \bar{\varepsilon}_\Lambda^2)^2 [\ln^2|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]}. \quad (102)$$

Both the function c , and the function \bar{c} may be shown to vanish at zero binding energy limit. So, we have the following property of these functions:

$$c(0) = \bar{c}(0) = 0.$$

Hence, it is only in the zero binding energy limit that we get an asymptotically free behavior of the function V_{AR} , coinciding with the behavior of the Richardson's potential at small distances. In the case of a nonzero binding energy, according to expressions (99) and (100), the following asymptotic representation will be valid for the function V_{AR} at small distances:

$$V_{AR}(\varepsilon, \Lambda; r) = \frac{\alpha(\varepsilon_\Lambda, \Lambda r)}{r}, \quad r \ll \frac{1}{\Lambda} < \frac{1}{|\varepsilon|}. \quad (103)$$

In this expression, the running coupling constant has an explicit energy dependence, and

$$\alpha(\varepsilon_\Lambda, \Lambda r)|_{\varepsilon=0} = \alpha_R(\Lambda r) = \frac{G\pi^2}{\ln(\frac{1}{\Lambda r})}. \quad (104)$$

Also,

$$\begin{aligned}\alpha(\varepsilon_\Lambda, \Lambda r)|_{r=0} &= G\pi^2 c(\varepsilon_\Lambda), \quad \varepsilon > 0, \\ &= G\pi^2 \bar{c}(\bar{\varepsilon}_\Lambda), \quad \varepsilon = -\bar{\varepsilon} < 0.\end{aligned}$$

The conclusion we are driven to is that when the binding energy is different from zero, the function V_{AR} , which is the local approximation of the two-quark interaction quasipotential in quantum chromodynamics, has a Coulomb singularity at zero.

7 Conclusion

In this paper the results of calculating the interaction quasipotential for two quarks in QCD by early developed single-time formalism in QFT for two-fermion systems have been presented. At the first step we obtained analytical expressions for the quark-antiquark interaction quasipotential in one-gluon exchange approximation which explicitly take into account the structure of the initial quantum field theory gauge model.

Then we have studied the influence of the infrared singularities of the gluon Green's functions on the behaviour of the interaction potential for two quarks in QCD. The singular behaviour of the gluon Green's function of the form κ^2/k^4 is known to be the result of nonperturbative investigations of the infrared region in QCD. Therefore the results we obtained may also be considered as going beyond the perturbative theory when calculating quark-quark forces in the framework of the fundamental QCD Lagrangian. Our consideration of the quark interaction problem shows that the generally accepted notions, that the singularity of the gluon propagator κ^2/k^4 corresponds to the interaction potential linearly growing with distance are not quite correct. As can easily be seen from formulae (51,53) the indicated correspondence is restored only at zero binding energy. Moreover, we have shown that when the binding energy is negative the infrared singularity of the gluon propagator does not lead to a potential linearly growing with distance. However when going over to the region of positive values for the binding energy there appear oscillations with the amplitude linearly increasing with distance. The question what such oscillations have to do with the confinement problem remains open. At the same time one should note, that the analogy with the solid state physics, where we find oscillating potentials, allows us to consider

such oscillations of forces as a manifestation of the quasicrystal structure of the vacuum in QFT.

As we have already noted, a consistent relativistic consideration of the two body problem in the framework of local QFT brings us to a nontrivial dependence of the interaction potential on energy. Such energy dependence assigns the interaction potential with rather unusual properties in the configuration space which could hardly be imagined if one stick to habitual quantum mechanical intuition. In particular, the energy dependence of the interaction potential results in the fact that the properties of the forces qualitatively change during the transition from the discrete spectrum to the region of the continuous one: a smooth behaviour of the interaction potential in the discrete spectrum is replaced by the oscillations in the continuous spectrum, and this change in the behaviour of the forces is universal, i.e. independent of concrete quantum field theory model and of used approximations. The causal structure of local quantum field theory displays in such manner. Recent studies [18] revealed an extremely interesting property of the oscillating potentials: such potentials lead to the appearance of discrete levels in the continuous spectrum.

Here we were also interested in the problem of the gauge dependence of the interaction potential. Here at least two observed facts seem to be rather important. First, there is a special gauge $d = -3$, where one manage correctly to describe the infrared region in terms of the interaction quasipotential. Only in this gauge the divergences are canceled, and we arrive at a finite result for the interaction quasipotential in the configuration space with an account of the infrared singularities of gluon propagator. Secondly, a very weak dependence of the interaction quasipotential on the gauge parameter $d^{(0)}$ in the discrete spectrum becomes essentially stronger when going over to the continuous spectrum, in this case in the continuous spectrum at large distances the interaction potential acquires "knot" points, which are invariant w.r.t. the choice of gauge. The position of the "knots" does not depend on the value of the gauge parameter $d^{(0)}$ and is determined only by the binding energy of the system.

We shall also point out a possibility for a new physical interpretation for the parameter κ , which is known not to be calculated in the original fundamental model, but appears as reflection of the widely discussed phenomenon of dimensional transmutation. In ref.[12] this parameter was determined phenomenologically from the slope of the linearly growing part of the interaction

potential with an account of the spectroscopy data. In our approach the parameter κ being the quantity of the same measure as the binding energy, fixes a new scale of distances determining the infrared region. It is remarkable that the quantity κ enters the interaction potential in the form of a dimensionless ratio $\beta \equiv (\kappa/\varepsilon)^2$, in this, explicit expressions (36) and (37) obviously show that the quantity β characterizes the intensity of the infrared region influence on the behaviour of the quark–quark forces.

In the present work we pursued the goal to elucidate how the property of asymptotic freedom from quantum chromodynamics manifests itself in terms of a quark–quark interaction quasipotential.

To achieve the goal, we have calculated the quark–quark interaction quasipotential applying the single–time reduction technique and obtained the corresponding explicit analytical expressions. In doing so, the one–loop approximation for the invariant charge in quantum chromodynamics was used.

The analysis of the resulting expressions for the interaction quasipotential has shown that here the same scenario is observed that the transition from the discrete spectrum region (negative values of the binding energy) to the continuous spectrum (region of positive values of the binding energy) changes the pattern in the behavior of quark–quark forces. In this manner the non-trivial energy dependence of the quark interaction quasipotential manifests itself. The obtained energy dependence of quark–quark forces, as it has been repeatedly stressed to be a consequence of a consistent consideration of the problem of interaction of relativistic systems.

There is one more situation described in this work where the energy dependence of quark–quark forces shows, and this one is related with the behavior of the running coupling constant in the configuration space at small distances. The behavior of the running coupling constant proves to be such that when the binding energy is not zero, the Coulomb singularity of the quark interaction quasipotential at zero is preserved, and it is only at zero binding energy limit this Coulomb singularity is logarithmically "smoothed over". Therefore, the known Richardson's potential and its relativized associations concerning the behavior of quark–quark forces are restored only in the limit at zero binding energy. It will of great interest to study the correspondence of the indicated properties of quark–quark forces and experimental data on spectroscopy and decays of quarkonium systems.

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Appendix A

Here we present the results of calculating of integrals which determine the functions $A^{(i)}$ and $B^{(i)}$, ($i = 0, 1$), in the case of unequal masses of quarks and antiquarks in arbitrary evolution gauge. The calculation of the integrals yields

$$\begin{aligned}
 A^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &\equiv A^{(1)}(M, P_\perp \mid p_\perp; k_\perp) = \\
 &= \frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{-4(p_\perp - k_\perp)^2} \times \\
 &\times \left[\frac{1}{(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^2} + \right. \\
 &+ \frac{1}{(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^2} + \\
 &+ \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})} + \\
 &\left. + \frac{1}{\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})} \right],
 \end{aligned}
 \tag{105}$$

$$\begin{aligned}
 B^{(1)}(nM \mid \tilde{p}_1 \tilde{p}_2; \tilde{k}_1 \tilde{k}_2) &\equiv B^{(1)}(M, P_\perp \mid p_\perp; k_\perp) = \\
 &= -\frac{(g\kappa)^2}{(2\pi)^3} \cdot \frac{1}{(2\sqrt{-(p_\perp - k_\perp)^2})^3} \times \\
 &\times \left\{ \frac{[\frac{1}{2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M]^2}{(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^3} + \right. \\
 &+ \frac{[\frac{1}{2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M]^2}{(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^3} +
 \end{aligned}
 \tag{106}$$

$$\begin{aligned}
& + \frac{3(\sqrt{m_1^2 - p_{1\perp}^2} - \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2})^2}{2\sqrt{-(p_\perp - k_\perp)^2}} \times \\
& \times \left[\frac{1}{2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M \right]^2 \times \\
& \quad \times [(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M) \times \\
& \quad \times (\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)]^{-2} - \\
& - 3 \left[1 - \frac{(\sqrt{m_1^2 - p_{1\perp}^2} - \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2})^2}{-4(p_\perp - k_\perp)^2} \right] \times \\
& \times \left[\frac{1}{2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M \right] \times \\
& \quad \times [(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M) \times \\
& \quad \times (\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)]^{-1} - \\
& - \frac{(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} - \sqrt{m_2^2 - k_{2\perp}^2})^2}{4} \times \\
& \quad \times \left[\frac{1}{(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^3} + \right. \\
& \quad \left. + \frac{1}{(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^3} + \right. \\
& \quad + \frac{3}{2\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^2} + \\
& \quad + \frac{3}{2\sqrt{-(p_\perp - k_\perp)^2}(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)^2} + \\
& \quad + \frac{3}{2(\sqrt{-(p_\perp - k_\perp)^2})^2(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)} + \\
& \quad \left. + \frac{3}{2(\sqrt{-(p_\perp - k_\perp)^2})^2(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)} \right] \Bigg\}.
\end{aligned}$$

We used here the following notations

$$\begin{aligned}
p_{i\perp} &= \tilde{p}_i - (n\tilde{p}_i)n, & k_{i\perp} &= \tilde{k}_i - (n\tilde{k}_i)n, & i &= 1, 2, \\
P_\perp &= p_{1\perp} + p_{2\perp}, & K_\perp &= k_{1\perp} + k_{2\perp}, & P_\perp &= K_\perp, \\
p_\perp &= p_{1\perp}, & p_{2\perp} &= P_\perp - p_\perp, & k_\perp &= k_{1\perp}, & k_{2\perp} &= P_\perp - k_\perp.
\end{aligned} \tag{107}$$

$$\begin{aligned}
A^{(0)}(nM \mid \tilde{p}_1\tilde{p}_2; \tilde{k}_1\tilde{k}_2) &\equiv A^{(0)}(M, P_\perp \mid p_\perp; k_\perp) = \\
&= \frac{g^2}{(2\pi)^3} \cdot \frac{1}{2\sqrt{-(p_\perp - k_\perp)^2}} \times
\end{aligned} \tag{108}$$

$$\begin{aligned}
&\times \left[\frac{1}{\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M} + \right. \\
&\left. + \frac{1}{\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M} \right],
\end{aligned}$$

$$B^{(0)}(nM \mid \tilde{p}_1\tilde{p}_2; \tilde{k}_1\tilde{k}_2) \equiv B^{(0)}(M, P_\perp \mid p_\perp; k_\perp) = \tag{109}$$

$$\begin{aligned}
&= \frac{-g^2}{(2\pi)^3} \cdot \left\{ \frac{1}{2} (\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \right. \\
&\quad \left. + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M \right]^2 \times \\
&\quad \times [(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M) \times \\
&\quad \times (\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M)]^{-2} \times \\
&\quad \times \frac{(\sqrt{m_1^2 - p_{1\perp}^2} - \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2})^2}{-4(p_\perp - k_\perp)^2} \Big] - \\
&\quad - \left[1 - \frac{(\sqrt{m_1^2 - p_{1\perp}^2} - \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2})^2}{-4(p_\perp - k_\perp)^2} \right] \times \\
&\quad \times \frac{\frac{1}{2}(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2}) - M}{2\sqrt{-(p_\perp - k_\perp)^2}} \times \\
&\quad \times [(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2} - M) \times
\end{aligned}$$

$$\begin{aligned}
& \times (\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^{-1} - \\
& - \frac{(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - p_{2\perp}^2} - \sqrt{m_1^2 - k_{1\perp}^2} - \sqrt{m_2^2 - k_{2\perp}^2})^2}{4} \times \\
& \times \left[\frac{1}{-4(p_\perp - k_\perp)^2(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^2} + \right. \\
& + \frac{1}{-4(p_\perp - k_\perp)^2(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})^2} + \\
& + \frac{1}{4(\sqrt{-(p_\perp - k_\perp)^2})^3(\sqrt{m_1^2 - p_{1\perp}^2} + \sqrt{m_2^2 - k_{2\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})} + \\
& \left. + \frac{1}{4(\sqrt{-(p_\perp - k_\perp)^2})^3(\sqrt{m_2^2 - p_{2\perp}^2} + \sqrt{m_1^2 - k_{1\perp}^2} + \sqrt{-(p_\perp - k_\perp)^2 - M})} \right] \Bigg\}
\end{aligned}$$

Formulae (108) and (109) use the same notations as (107).

Appendix B

In the present Appendix, the integrals (66), (73), (74) from the main text are calculated for the case of different quark masses and in an arbitrary evolution gauge.

1. We start with integral (66) determining the A function. This integral is written down in the form

$$I_A = \int_C dz f_A(z) \quad (110)$$

with

$$f_A(z) = \left(\frac{1}{\delta + z} + \frac{1}{\delta - z} \right) \left(\frac{1}{\kappa_1 - z} + \frac{1}{z - \kappa_2} \right) \frac{1}{\ln[(\kappa'_1 - z)(z - \kappa'_2)/\Lambda^2]}. \quad (111)$$

The quantities $\delta, \kappa_{1,2}, \kappa'_{1,2}$ are defined in the main text, and the path of integration C is depicted in Fig. 1. The same figure shows the analytic structure

of the function $f_A(z)$: the function f_A has the logarithmic branch points $z = \kappa'_{1,2}$ and the poles at the points $z = (\pm\delta, \kappa_{1,2}, x_{1,2})$, where

$$x_{1,2} = (np - nk) \pm \sqrt{(\xi - 1)\Lambda^2 - (p - k)_\perp^2}.$$

The path of integration C in expression (110) for the function I_A is determined by bypass rules for the singularities given in integral (66), which are unambiguously derived from the causal structure of the local quantum field theory. Integral (110) is calculated by closing the path of integration in either the upper, or the lower half-plane. When the path of integration is closed in the upper half-plane, there arises an integral due to the discontinuity of the integrand function on the left cut (see Fig. 1). In this case, we have that

$$\int_C dz f_A(z) = 2\pi i \sum_L \text{Res} f_A(z_L) - \int_{-\infty}^{\kappa'_2} dx \Delta_L f_A(x), \quad (112)$$

where $\Delta_L f_A$ means the discontinuity of the function f_A on the left cut

$$\Delta_L f(x) = f(x + i0) - f(x - i0), \quad x < \kappa'_2,$$

and the residues are taken at the poles $z_L = (-\delta, \kappa_2, x_2)$. Calculating the discontinuity of the function f_A on the left cut and the residues of the same function at the given poles, for the quantity I_A we get

$$\begin{aligned} I_A = & -2\pi i (\kappa_1 - \kappa_2) \left[\frac{1}{(\kappa_1 + \delta)(\kappa_2 + \delta) \ln[-(\kappa'_1 + \delta)(\kappa'_2 + \delta)/\Lambda^2]} - \right. \\ & - \frac{1}{\ln \xi} \cdot \frac{2\delta}{(\kappa_1 - \kappa_2)(\delta^2 - \kappa_2^2)} - \frac{2\delta \Lambda^2}{(x_1 - x_2)(\kappa_1 - x_2)(x_2 - \kappa_2)(\delta^2 - x_2^2)} - \\ & \left. - \int_{-\infty}^{\kappa'_2} dx \frac{2\delta}{(\kappa_1 - x)(x - \kappa_2)(\delta^2 - x^2) [\ln^2 |(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} \right]. \quad (113) \end{aligned}$$

When deriving expression (113), we did allow for the equality

$$(\kappa'_1 - \kappa_2)(\kappa_2 - \kappa'_2) = \xi \Lambda^2. \quad (114)$$

Had we calculated integral (110) by closing the path of integration in the lower half-plane, then instead of (112) we would have obtained

$$\int_C dz f_A(z) = -2\pi i \sum_R \text{Res} f_A(z_R) + \int_{\kappa'_1}^{\infty} dx \Delta_R f_A(x)$$

with $\Delta_R f_A$ meaning the discontinuity of the function f_A on the right cut

$$\Delta_R f_A(x) = f(x + i0) - f(x - i0), \quad x > \kappa'_1.$$

The residues in this case are taken at the poles $z_R = (\delta, \kappa_1, x_1)$. After the corresponding calculations, we obtain the following expression for the quantity I_A :

$$\begin{aligned} I_A = & -2\pi i(\kappa_1 - \kappa_2) \left[\frac{1}{(\delta - \kappa_1)(\delta - \kappa_2) \ln[-(\delta - \kappa'_1)(\delta - \kappa'_2)/\Lambda^2]} - \right. \\ & - \frac{1}{\ln \xi} \cdot \frac{2\delta}{(\kappa_1 - \kappa_2)(\delta^2 - \kappa_1^2)} - \frac{2\delta\Lambda^2}{(x_1 - x_2)(\kappa_1 - x_1)(x_1 - \kappa_2)(\delta^2 - x_1^2)} - \\ & \left. - \int_{\kappa'_1}^{\infty} dx \frac{2\delta}{(\kappa_1 - x)(x - \kappa_2)(\delta^2 - x^2)[\ln^2|(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} \right]. \quad (115) \end{aligned}$$

Here we take into account that

$$(\kappa'_1 - \kappa_1)(\kappa_1 - \kappa'_2) = \xi\Lambda^2. \quad (116)$$

Expression (115) for I_A is derived from expression (113) through the substitution $\kappa_1 \rightarrow -\kappa_2, \kappa_2 \rightarrow -\kappa_1, \kappa'_1 \rightarrow -\kappa'_2, \kappa'_2 \rightarrow -\kappa'_1$. As a consequence, the following substitution takes place: $x_1 \rightarrow -x_2, x_2 \rightarrow -x_1$. As can easily be seen, integral (110) for I_A remains invariant under the substitution. Therefore, (113) and (115) yield two equivalent representations for the quantity I_A . By taking the half-sum of expressions (113) and (115), we can get a symmetric representation for the quantity I_A .

If quark masses are assumed to be equal, and a particular evolution gauge (Markov–Yukawa gauge) is chosen, then, remembering that in this case

$$\begin{aligned} \kappa_1 = -\kappa_2 & \equiv \kappa_0 = \sqrt{-(p - k)_\perp^2} \equiv q, \\ \kappa'_1 = -\kappa'_2 & \equiv \kappa'_0 = \sqrt{\xi\Lambda^2 + q^2}, \\ x_1 = -x_2 & \equiv x_0 = \sqrt{(\xi - 1)\Lambda^2 + q^2}, \end{aligned} \quad (117)$$

from (113) or (115) we obtain

$$I_A = -2\pi i \cdot 2\kappa_0 \left[\frac{1}{(\delta^2 - \kappa_0^2) \ln[(\kappa'_0)^2 - \delta^2]/\Lambda^2]} - \right.$$

$$\begin{aligned}
& -\frac{1}{\ln \xi} \cdot \frac{\delta}{\kappa_0(\delta^2 - \kappa_0^2)} + \frac{\delta}{(\xi - 1)x_0(\delta^2 - x_0^2)} - \\
& - \int_{\kappa'_0}^{\infty} dx \frac{2\delta}{(x^2 - \delta^2)(x^2 - \kappa_0^2)} \cdot \frac{1}{\ln^2[(x^2 - \kappa_0'^2)/\Lambda^2] + \pi^2} \Big]. \quad (118)
\end{aligned}$$

Substituting the integration variable $x^2 - q^2 = \Lambda^2 y^2$ in the R.H.S. of Eq. (118) and taking into account expression (117) for the quantities κ_0, κ'_0, x_0 , from (118) we get representation (68) for the function A .

2. Write integral (73), specifying the function $B^{(1)}$, in the form analogous to (110)

$$I_B^{(1)} = \int_C dz f_B^{(1)}(z), \quad (119)$$

where

$$f_B^{(1)}(z) = \left(\frac{1}{\delta + z} + \frac{1}{\delta - z} \right) \left(\frac{1}{\kappa_1 - z} + \frac{1}{z - \kappa_2} \right)^2 \frac{x^2}{\ln[(\kappa'_1 - z)(z - \kappa'_2)/\Lambda^2]}. \quad (120)$$

In the z plane, we integrate over the same path C as depicted in Fig. 1. The function $f_B^{(1)}$ has the logarithmic branch points $z = \kappa'_{1,2}$, simple poles at $z = (\pm\delta, x_{1,2})$, and the poles of order 2 at the points $z = \kappa_{1,2}$. As before, integral (119) is calculated by closing the path of integration in the upper or lower half-plane, after which the residues at the corresponding poles and discontinuities of the function $f_b^{(1)}$ on the corresponding cuts are found. The final result for the quantity $I_B^{(1)}$ is given in the symmetric form

$$\begin{aligned}
I_B^{(1)} = 2\pi i(\kappa_1 - \kappa_2) & \left[-\frac{\delta^2}{(\delta + \kappa_1)^2(\delta + \kappa_2)\ln[-(\delta + \kappa'_1)(\delta + \kappa'_2)/\Lambda^2]} - \right. \\
& - \frac{\delta^2}{(\delta - \kappa_2)^2(\delta - \kappa_1)\ln[-(\delta - \kappa'_1)(\delta - \kappa'_2)/\Lambda^2]} + \\
& + \frac{1}{\ln \xi} \cdot \frac{2\delta}{(\kappa_1 - \kappa_2)^2} \left(\frac{\kappa_1^2}{\delta^2 - \kappa_1^2} + \frac{\kappa_2^2}{\delta^2 - \kappa_2^2} \right) + \frac{\Lambda^2}{x_1 - x_2} \times \\
& \times \left(\frac{2\delta x_2^2}{(\delta^2 - x_2^2)(\kappa_1 - x_2)^2(x_2 - \kappa_2)} + \frac{2\delta x_1^2}{(\delta^2 - x_1^2)(x_1 - \kappa_2)^2(\kappa_1 - x_1)} \right) + \\
& + \int_{-\infty}^{\kappa'_2} dx \frac{2\delta x^2}{(\delta^2 - x^2)(\kappa_1 - x)^2(x - \kappa_2)[\ln^2|(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} +
\end{aligned}$$

$$+ \int_{\kappa'_1}^{\infty} dx \frac{2\delta x^2}{(\delta^2 - x^2)(\kappa_1 - x)(x - \kappa_2)^2 [ln^2 |(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} \Big] \quad (121)$$

Provided the quark masses are equal, in the above-mentioned particular evolution gauge we come to

$$\begin{aligned} I_B^{(1)} = 2\pi i \cdot 2\kappa_0 & \left[-\frac{2\delta^2}{(\delta + \kappa_0)(\delta^2 - \kappa_0^2)ln[(\kappa_0'^2 - \delta^2)/\Lambda^2]} + \right. \\ & \left. + \frac{1}{ln\xi} \cdot \frac{\delta}{\delta^2 - \kappa_0^2} - \frac{2\delta x_0}{(\xi - 1)(\delta^2 - x_0^2)(\kappa_0 + x_0)} + \right. \\ & \left. + \int_{\kappa'_0}^{\infty} dx \frac{4\delta x^2}{(x^2 - \delta^2)(x^2 - \kappa_0^2)(x + \kappa_0)} \cdot \frac{1}{ln^2[(x^2 - \kappa_0'^2)/\Lambda^2] + \pi^2} \right]. \quad (122) \end{aligned}$$

Representation (75) for the function $B^{(1)}$ follows now from equality (122) after the integration variable therein is replaced as $x^2 - q^2 = \Lambda^2 y^2$ and the explicit expressions (117) for κ_0, x_0 are taken into account.

3. Writing integral (74), which determines the function $B^{(2)}$, in the form

$$I_B^{(2)} = \int_C dz f_B^{(2)}(z), \quad (123)$$

with

$$f_B^{(2)}(z) = \left(\frac{1}{\delta + z} + \frac{1}{\delta - z} \right) \left(\frac{1}{\kappa_1 - z} + \frac{1}{z - \kappa_2} \right)^2 \frac{1}{ln[(\kappa'_1 - z)(z - \kappa'_2)/\Lambda^2]}, \quad (124)$$

and repeating the above calculations, for the quantity $I_B^{(2)}$ we get that

$$\begin{aligned} I_B^{(2)} = 2\pi i(\kappa_1 - \kappa_2) & \left[-\frac{1}{(\delta + \kappa_1)^2(\delta + \kappa_2)ln[-(\delta + \kappa'_1)(\delta + \kappa'_2)/\Lambda^2]} - \right. \\ & \left. - \frac{1}{(\delta - \kappa_2)^2(\delta - \kappa_1)ln[-(\delta - \kappa'_1)(\delta - \kappa'_2)/\Lambda^2]} + \right. \\ & \left. + \frac{1}{ln\xi} \cdot \frac{2\delta}{(\kappa_1 - \kappa_2)^2} \left(\frac{1}{\delta^2 - \kappa_1^2} + \frac{1}{\delta^2 - \kappa_2^2} \right) + \frac{2\delta\Lambda^2}{x_1 - x_2} \times \right. \\ & \left. \times \left(\frac{1}{(\delta^2 - x_2^2)(\kappa_1 - x_2)^2(x_2 - \kappa_2)} + \frac{1}{(\delta^2 - x_1^2)(x_1 - \kappa_2)^2(\kappa_1 - x_1)} \right) + \right. \\ & \left. + \int_{-\infty}^{\kappa'_2} dx \frac{2\delta}{(\delta^2 - x^2)(\kappa_1 - x)^2(x - \kappa_2)[ln^2 |(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} + \right. \end{aligned}$$

$$+ \int_{\kappa'_1}^{\infty} dx \frac{2\delta}{(\delta^2 - x^2)(\kappa_1 - x)(x - \kappa_2)^2 [\ln^2 |(\kappa'_1 - x)(x - \kappa'_2)/\Lambda^2| + \pi^2]} \Big] \quad (125)$$

If in the resulting expression (125) we put quark masses to be equal and pick up Markov–Yukawa evolution gauge, then it follows that

$$\begin{aligned} I_B^{(2)} = & 2\pi i \cdot 2\kappa_0 \left[-\frac{2}{(\delta + \kappa_0)(\delta^2 - \kappa_0^2) \ln[(\kappa'_0{}^2 - \delta^2)/\Lambda^2]} + \right. \\ & + \frac{1}{\ln \xi} \cdot \frac{\delta}{\kappa_0^2(\delta^2 - \kappa_0^2)} - \frac{2\delta}{(\xi - 1)x_0(\delta^2 - x_0^2)(\kappa_0 + x_0)} + \\ & \left. + \int_{\kappa'_0}^{\infty} dx \frac{4\delta}{(x^2 - \delta^2)(x^2 - \kappa_0^2)(x + \kappa_0)} \cdot \frac{1}{\ln^2[(x^2 - \kappa'_0{}^2)/\Lambda^2] + \pi^2} \right]. \quad (126) \end{aligned}$$

Finally, the above substitution of the integration variable in the integral of the R.H.S. of Eq. (126) and the account of the explicit expressions (117) for the quantities κ_0, x_0 bring us to representation (77) for the function $B^{(2)}$.

Appendix C

When one goes over to the configuration space, one has to calculate the integrals of the form

$$\tilde{F}(\varepsilon, \Lambda; r) = \int_0^{\infty} q dq \sin(qr) F(\varepsilon, \Lambda; q). \quad (127)$$

Since in the momentum representations the functions we are working with are dependent on the dimensionless variables

$$F(\varepsilon, \Lambda; q) = \frac{1}{\Lambda^2} f(\varepsilon_{\Lambda}; q_{\Lambda}), \quad (128)$$

the integral transformation (127) is also rewrite in terms of the dimensionless variables

$$\tilde{f}(\varepsilon_{\Lambda}; \Lambda r) = \int_0^{\infty} q_{\Lambda} dq_{\Lambda} \sin(\Lambda r q_{\Lambda}) f(\varepsilon_{\Lambda}; q_{\Lambda}). \quad (129)$$

1. The calculation of integral (86) for the function A_R reduces to calculating five integrals of the form (129) for the five functions, corresponding to the five terms in the R.H.S. of equality (85). Let us write down these five functions:

$$f_A^{(1)}(\varepsilon_{\Lambda}; q_{\Lambda}) = \frac{1}{[q_{\Lambda}^2 - (\varepsilon_{\Lambda} + i0)^2] \ln[1 + q_{\Lambda}^2 - (\varepsilon_{\Lambda} + i0)^2]},$$

$$\begin{aligned}
f_A^{(2)}(\varepsilon_\Lambda; q_\Lambda) &= \frac{1}{q_\Lambda[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]}, & f_A^{(3)}(\varepsilon_\Lambda; q_\Lambda) &= \frac{1}{q_\Lambda^3[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]}, \\
f_A^{(4)}(\varepsilon_\Lambda; q_\Lambda) &= \frac{1}{q_\Lambda[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]^2}, \\
f_A^{(5)}(\varepsilon_\Lambda; q_\Lambda) &= \frac{1}{\sqrt{y^2 + q_\Lambda^2}[y^2 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]}. \tag{130}
\end{aligned}$$

Integral (129) of the function $f_A^{(1)}$ may be rewritten in the form

$$\tilde{f}_A^{(1)}(\varepsilon_\Lambda; \Lambda r) = \frac{1}{2i} \int_{-\infty}^{\infty} q_\Lambda dq_\Lambda \frac{\exp(i\Lambda r q_\Lambda)}{[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2] \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]}. \tag{131}$$

Now we represent it as a path integral

$$\tilde{f}_A^{(1)}(\varepsilon_\Lambda; \Lambda r) = \frac{1}{2i} \int_C dz g_A^{(1)}(\varepsilon_\Lambda; \Lambda r, z), \tag{132}$$

where

$$g_A^{(1)}(\varepsilon_\Lambda; \Lambda r, z) = \frac{z \exp(i\Lambda r z)}{(z^2 - \varepsilon_\Lambda^2) \ln(1 + z^2 - \varepsilon_\Lambda^2)}, \tag{133}$$

and the path of integration is shown in Fig. 2.

Suppose that $0 < \varepsilon_\Lambda < 1$. Then the analytic structure of the function $g_A^{(1)}$ is as in Fig. 2: the function $g_A^{(1)}$ has the logarithmic branch points $z = \pm i\sqrt{1 - \varepsilon_\Lambda^2}$ and the poles of order 2 at the points $z = \pm \varepsilon_\Lambda$. Integral (132) is calculated by closing the path of integration in the upper half-plane. As a result, we get

$$\begin{aligned}
\tilde{f}_A^{(1)}(\varepsilon_\Lambda; \Lambda r) &= \frac{\pi}{4} e^{i\Lambda r \varepsilon_\Lambda} \left(1 + \frac{i\Lambda r}{\varepsilon_\Lambda}\right) - \\
&- \pi \int_{\sqrt{1-\varepsilon_\Lambda^2}}^{\infty} y dy \frac{\exp(-\Lambda r y)}{(y^2 + \varepsilon_\Lambda^2) [\ln^2(y^2 + \varepsilon_\Lambda^2 - 1) + \pi^2]}. \tag{134}
\end{aligned}$$

Were the binding energy is negative: $\varepsilon_\Lambda = -\bar{\varepsilon}_\Lambda < 0, 0 < \bar{\varepsilon}_\Lambda < 1$, the result would have been

$$\tilde{f}_A^{(1)}(\varepsilon_\Lambda; \Lambda r) = \frac{\pi}{4} e^{-i\Lambda r \bar{\varepsilon}_\Lambda} \left(1 - \frac{i\Lambda r}{\bar{\varepsilon}_\Lambda}\right) -$$

$$-\pi \int_{\sqrt{1-\varepsilon_\Lambda^2}}^{\infty} y dy \frac{\exp(-\Lambda r y)}{(y^2 + \varepsilon_\Lambda^2)[\ln^2(y^2 + \varepsilon_\Lambda^2 - 1) + \pi^2]}.$$

Hence, the result (134) is true for the region $|\varepsilon_\Lambda| < 1$. The transition to the region $|\varepsilon_\Lambda| > 1$ is given by the analytic continuation of expression (134). It is possible to replace the integration variable in (134): $y^2 + \varepsilon_\Lambda^2 = x^2$, bringing it thus to the form

$$\int_1^{\infty} \frac{dy}{y} \frac{\exp(-\Lambda r \sqrt{y^2 - \varepsilon_\Lambda^2})}{\ln^2(y^2 - 1) + \pi^2},$$

which is, really, applied in formula (87).

The integrals like (129) for the functions $f_A^{(2,3,4)}$ can easily be reduced to those from the Tables. So, here only the results are given.

$$\tilde{f}_A^{(2)}(\varepsilon_\Lambda; \Lambda r) = \frac{1}{\varepsilon_\Lambda} \left[\frac{\pi}{2} e^{i\Lambda r \varepsilon_\Lambda} - a(\Lambda r \varepsilon_\Lambda) \right],$$

$$\tilde{f}_A^{(3)}(\varepsilon_\Lambda; \Lambda r) = \frac{1}{\varepsilon_\Lambda^2} \left\{ \frac{1}{\varepsilon_\Lambda} \left[\frac{\pi}{2} e^{i\Lambda r \varepsilon_\Lambda} - a(\Lambda r \varepsilon_\Lambda) \right] - \frac{\pi}{2} \Lambda r \left[i + \frac{2}{\pi} b(\mu r) \right] \Big|_{\mu \rightarrow 0} \right\},$$

$$\begin{aligned} \tilde{f}_A^{(4)}(\varepsilon_\Lambda; \Lambda r) = \\ = \frac{1}{2\varepsilon_\Lambda} \left\{ -\frac{1}{\varepsilon_\Lambda^2} \left[\frac{\pi}{2} e^{i\Lambda r \varepsilon_\Lambda} - a(\Lambda r \varepsilon_\Lambda) \right] + \frac{\Lambda r}{\varepsilon_\Lambda} \left[i \frac{\pi}{2} e^{i\Lambda r \varepsilon_\Lambda} + b(\Lambda r \varepsilon_\Lambda) \right] \right\} \end{aligned} \quad (135)$$

with the functions $a(x)$ and $b(x)$ defined in the main text. Expressions (135), which were obtained for positive values of the binding energy, hold also for its negative values. To verify this, one has to keep in mind that the functions a and b have logarithmic cuts, and the values of these functions at the cuts are given by the relations

$$a(e^{\pm i\pi} x) = -a(x) + \pi e^{\mp i x}, \quad b(e^{\pm i\pi} x) = b(x) \mp i\pi e^{\mp i x}, \quad x > 0.$$

Integral (129) for the function $f_A^{(5)}$ is rewritten in the form of (131)

$$\tilde{f}_A^{(5)}(\varepsilon_\Lambda; \Lambda r) = \frac{1}{2i} \int_{-\infty}^{\infty} z dz \frac{\exp(i\Lambda r z)}{\sqrt{y^2 + z^2} (y^2 + z^2 - \varepsilon_\Lambda^2)}. \quad (136)$$

The analytic structure of the integrand function in (136) can be seen in Fig. 3: there are root type branchings at the points $z = \pm i y$ and simple

poles at $z = \pm i\sqrt{y^2 - \varepsilon_\Lambda^2}$. After closing the path of integration in the upper half-plane, we find that

$$\tilde{f}_A^{(5)}(\varepsilon_\Lambda; \Lambda r) = \frac{\pi}{2|\varepsilon_\Lambda|} e^{-\Lambda r \sqrt{y^2 - \varepsilon_\Lambda^2}} - \int_y^\infty \frac{dx x \exp(-\Lambda r x)}{\sqrt{x^2 - y^2}(x^2 - y^2 + \varepsilon_\Lambda^2)}. \quad (137)$$

In the obtained expression (137), one can make the substitution of the integration variable: $x^2 - y^2 = t^2$, bringing it thus to the form

$$\int_0^\infty dt \frac{\exp(-\Lambda r \sqrt{t^2 + y^2})}{t^2 + \varepsilon_\Lambda^2}.$$

The calculations of the latter with the help of the residue theory technique yield

$$\begin{aligned} & \int_0^\infty dt \frac{\exp(-\Lambda r \sqrt{t^2 + y^2})}{t^2 + \varepsilon_\Lambda^2} = \\ & = \frac{\pi \exp(-\Lambda r \sqrt{y^2 - \varepsilon_\Lambda^2})}{2|\varepsilon_\Lambda|} - \int_0^\infty \frac{t dt \sin(\Lambda r t)}{\sqrt{t^2 + y^2}(t^2 + y^2 - \varepsilon_\Lambda^2)} \end{aligned}$$

which, allowing for (137), brings us back to the initial representation (129) for the function $\tilde{f}_A^{(5)}$. That is why we did not use result (137) in the bulk of the text.

2. The calculation of integral (86) for the function B_R also reduces to the calculation of five integrals of the form (129) for the five functions, corresponding to the five terms in the R.H.S. of equality (89). For the reasons just mentioned, we will not integrate the fifth term of (89) but leave it unchanged in the initial form. The functions, corresponding to the second, third and fourth terms of (89), coincide with the functions $f_A^{(2,3,4)}$. The result of integrating such functions has already been given. So, the only thing to investigate is integral (129) for the function, corresponding to the first term in the R.H.S. of Eq.(89)

$$f_B^{(1)}(\varepsilon_\Lambda; q_\Lambda) = \frac{1}{q_\Lambda(q_\Lambda - \varepsilon_\Lambda - i0)[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2] \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]}. \quad (138)$$

Here from the very beginning one should distinguish between the regions of positive and negative values of the binding energy.

a) The region of positive binding energy values: $\varepsilon_\Lambda > 0$. In this case, integral (129) for the function $f_B^{(1)}$ is conveniently written in the form

$$\begin{aligned} \tilde{f}_B^{(1)}(\varepsilon_\Lambda; \Lambda r) &= \\ &= \frac{1}{2i} \int_{-\infty}^{\infty} dq_\Lambda \frac{\exp(i\Lambda r q_\Lambda)}{[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2](q_\Lambda - \varepsilon_\Lambda - i0) \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} - \\ &- \frac{1}{2i} \int_{-\infty}^0 dq_\Lambda \frac{2\varepsilon_\Lambda \exp(i\Lambda r q_\Lambda)}{[q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]^2 \ln[1 + q_\Lambda^2 - (\varepsilon_\Lambda + i0)^2]} \equiv I_1 - I_2. \end{aligned} \quad (139)$$

As in the previous case, we rewrite the integral I_1 as a path integral with the path of integration shown in Fig. 2; the integrand function has a pole of order 3 at the point $z = \varepsilon_\Lambda$. After closing the integration path in the upper half-plane and calculating the residue of the integrand function at this pole and the discontinuity of the same function on the upper cut, we obtain that

$$\begin{aligned} I_1 &= -\frac{\pi e^{i\Lambda r \varepsilon_\Lambda}}{8\varepsilon_\Lambda^2} \left[1 + \frac{2}{3}\varepsilon_\Lambda^2 - \frac{3}{2\varepsilon_\Lambda^2} + 2i\Lambda r \left(\frac{1}{\varepsilon_\Lambda} - \varepsilon_\Lambda \right) + (\Lambda r)^2 \right] + \\ &+ \pi \int_{\sqrt{1-\varepsilon_\Lambda^2}}^{\infty} dt \frac{\exp(-\Lambda r t)}{(t^2 + \varepsilon_\Lambda^2)(t + i\varepsilon_\Lambda)[\ln^2(t^2 + \varepsilon_\Lambda^2 - 1) + \pi^2]}. \end{aligned} \quad (140)$$

The integral I_2 is readily brought to the form

$$I_2 = - \int_0^{\infty} dt \frac{\varepsilon_\Lambda \exp(-\Lambda r t)}{(t^2 + \varepsilon_\Lambda^2)^2 [\ln|1 - \varepsilon_\Lambda^2 - t^2| - i\pi\Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}. \quad (141)$$

Combining now (140) and (141), we derive the following expression for the function $\tilde{f}_B^{(1)}$:

$$\begin{aligned} \tilde{f}_B^{(1)}(\varepsilon_\Lambda; \Lambda r) &= \\ &= -\frac{\pi e^{i\Lambda r \varepsilon_\Lambda}}{8\varepsilon_\Lambda^2} \left[1 + \frac{2}{3}\varepsilon_\Lambda^2 - \frac{3}{2\varepsilon_\Lambda^2} + 2i\Lambda r \left(\frac{1}{\varepsilon_\Lambda} - \varepsilon_\Lambda \right) + (\Lambda r)^2 \right] + \\ &+ \int_0^{\infty} dt \frac{e^{-\Lambda r t} [\varepsilon_\Lambda \ln|1 - \varepsilon_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}{(t^2 + \varepsilon_\Lambda^2)^2 [\ln^2|1 - \varepsilon_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \varepsilon_\Lambda^2 - 1)]}. \end{aligned} \quad (142)$$

b) The region of negative values of the binding energy: $\varepsilon_\Lambda = -\bar{\varepsilon}_\Lambda < 0$, $\bar{\varepsilon}_\Lambda > 0$. Integral (129) for the function $\tilde{f}_B^{(1)}$ in this case is also representable

as the difference of the two integrals:

$$\tilde{f}_B^{(1)}(\varepsilon_\Lambda; \Lambda r) = I_1(\varepsilon_\Lambda; \Lambda r) - I_2(\varepsilon_\Lambda; \Lambda r),$$

where

$$\begin{aligned} I_1(\varepsilon_\Lambda; \Lambda r) &= \\ &= \frac{1}{2i} \int_{-\infty}^{\infty} dq_\Lambda \frac{\exp(i\Lambda r q_\Lambda)}{[q_\Lambda^2 - (\bar{\varepsilon}_\Lambda - i0)^2](q_\Lambda - \bar{\varepsilon}_\Lambda + i0) \ln[1 + q_\Lambda^2 - (\bar{\varepsilon}_\Lambda - i0)^2]}, \\ I_2(\varepsilon_\Lambda; \Lambda r) &= \frac{1}{2i} \int_0^\infty dq_\Lambda \frac{2\bar{\varepsilon}_\Lambda \exp(i\Lambda r q_\Lambda)}{[q_\Lambda^2 - (\bar{\varepsilon}_\Lambda - i0)^2]^2 \ln[1 + q_\Lambda^2 - (\bar{\varepsilon}_\Lambda - i0)^2]}. \end{aligned}$$

We recast the expression for $I_1(\varepsilon_\Lambda; \Lambda r)$ as the path integral

$$I_1(\varepsilon_\Lambda; \Lambda r) = \frac{1}{2i} \int_{\bar{C}} dz J_1(\bar{\varepsilon}_\Lambda; \Lambda r, z),$$

where

$$J_1(\bar{\varepsilon}_\Lambda; \Lambda r, z) = \frac{\exp(i\Lambda r z)}{(z^2 - \bar{\varepsilon}_\Lambda^2)(z - \bar{\varepsilon}_\Lambda) \ln(1 + z^2 - \bar{\varepsilon}_\Lambda^2)},$$

whereas the path of integration \bar{C} is presented in Fig. 4. The function $J_1(\bar{\varepsilon}_\Lambda; \Lambda r, z)$ has a pole of order 3 at the point $z = \bar{\varepsilon}_\Lambda$, a pole of order 2 at $z = -\bar{\varepsilon}_\Lambda$, and logarithmic branchings at the points $z = \pm i\sqrt{1 - \bar{\varepsilon}_\Lambda^2}$. Closing the path of integration in the upper half-plane and calculating the residue of the integrand function at the pole $z = -\bar{\varepsilon}_\Lambda$ and the discontinuity of this function on the upper cut, for I_1 we get that

$$\begin{aligned} I_1(\varepsilon_\Lambda; \Lambda r) &= \frac{\pi e^{-i\Lambda r \bar{\varepsilon}_\Lambda}}{8\bar{\varepsilon}_\Lambda^2} \left(1 - \frac{3}{2\bar{\varepsilon}_\Lambda^2} - \frac{i\Lambda r}{\bar{\varepsilon}_\Lambda} \right) + \\ &+ \pi \int_{\sqrt{1-\bar{\varepsilon}_\Lambda^2}}^\infty dt \frac{\exp(-\Lambda r t)}{(t^2 + \bar{\varepsilon}_\Lambda^2)(t + i\bar{\varepsilon}_\Lambda)[\ln^2(t^2 + \bar{\varepsilon}_\Lambda^2 - 1) + \pi^2]}. \end{aligned} \quad (143)$$

The integral for I_2 is easy reduce to the form

$$I_2(\varepsilon_\Lambda; \Lambda r) = \int_0^\infty dt \frac{\bar{\varepsilon}_\Lambda \exp(-\Lambda r t)}{(t^2 + \bar{\varepsilon}_\Lambda^2)^2 [\ln|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + i\pi\Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]}. \quad (144)$$

After that, combining the two expressions — (143) and (144)— we obtain the final result for the function $f_B^{(1)}$ to be

$$\begin{aligned} \tilde{f}_B^{(1)}(\varepsilon_\Lambda; \Lambda r) &= \frac{\pi e^{-i\Lambda r \bar{\varepsilon}_\Lambda}}{8\bar{\varepsilon}_\Lambda^2} \left(1 - \frac{3}{2\bar{\varepsilon}_\Lambda^2} - \frac{i\Lambda r}{\bar{\varepsilon}_\Lambda} \right) + \\ &+ \int_0^\infty dt \frac{e^{-\Lambda r t} [-\bar{\varepsilon}_\Lambda \ln|1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi t \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]}{(t^2 + \bar{\varepsilon}_\Lambda^2)^2 [l n^2 |1 - \bar{\varepsilon}_\Lambda^2 - t^2| + \pi^2 \Theta(t^2 + \bar{\varepsilon}_\Lambda^2 - 1)]} \end{aligned} \quad (145)$$

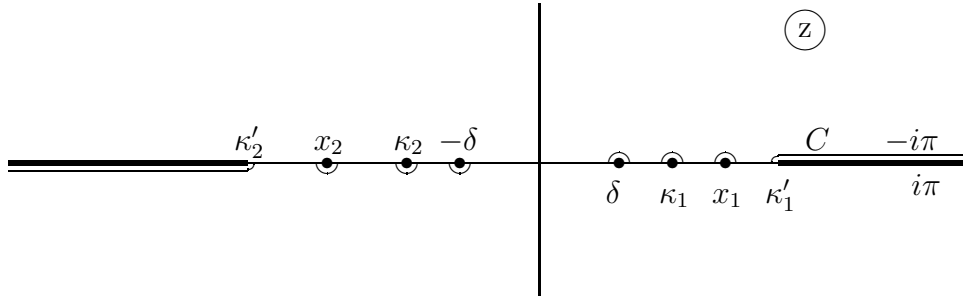


Fig. 1

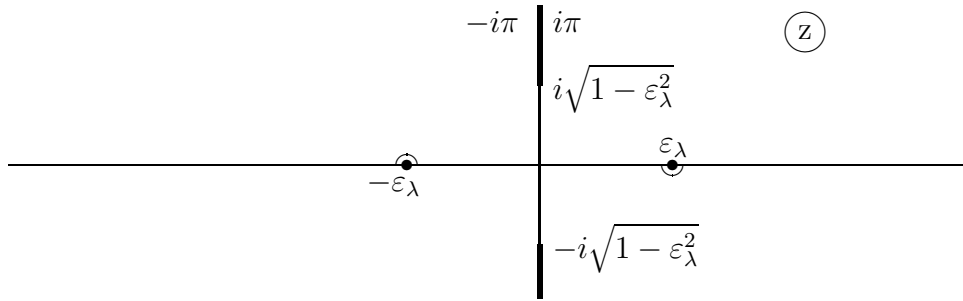


Fig. 2

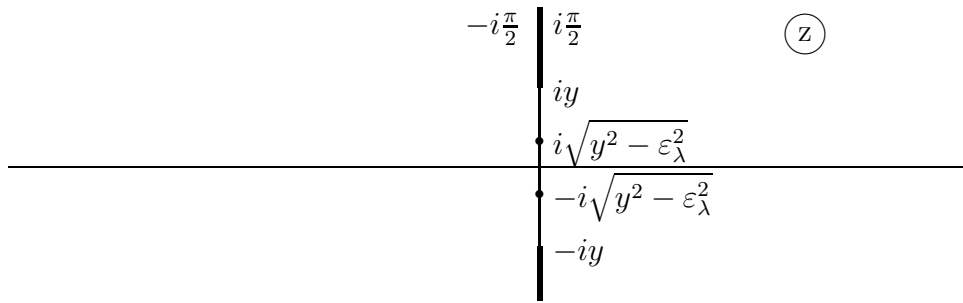


Fig. 3

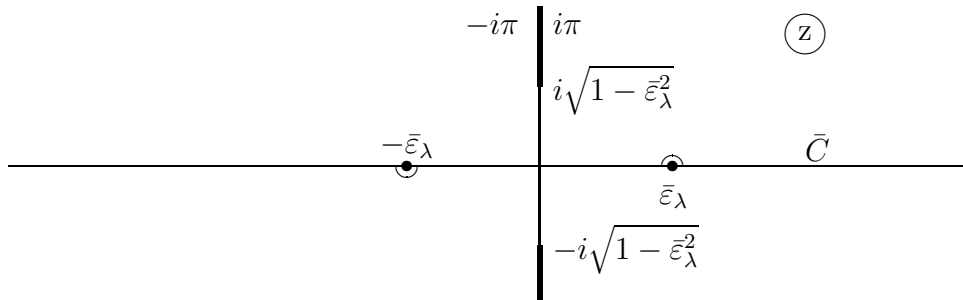


Fig. 4

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